MHD in a Cylindrical Shearing Box

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Suzuki, Taki, & Suriano, PASJ in press (arxiv:1904.05032)

Thanks to XC40@YITP & ATERUI@CfCA/NaOJ

Global MHD Simulation of a Disk

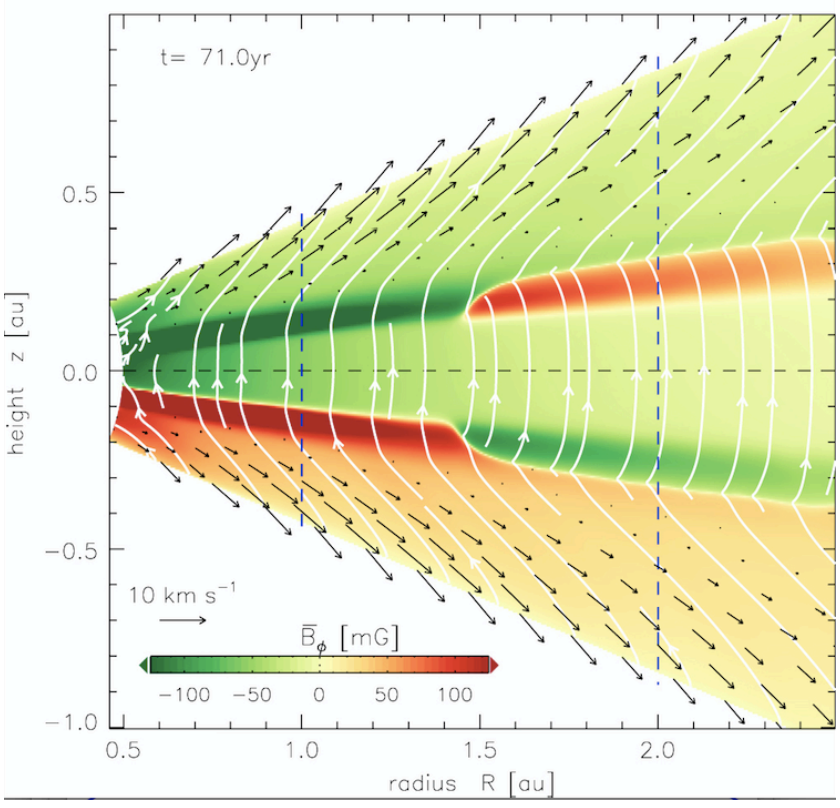
Suzuki & Inutsuka 2014

see also Hawley+ 2000; Beckwith+ 2009; Flock+ 2011; Parkin & Biknell 2013;

Takasao+ 2018; Zhu & Stone 2018 ...

Global Simulations with Non-ideal MHD

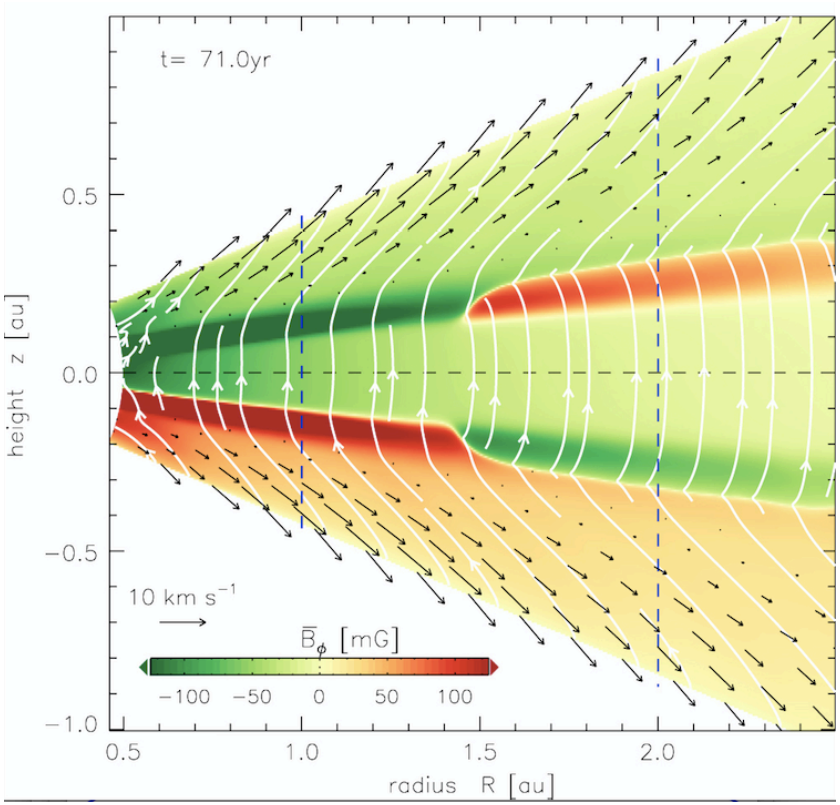
Global Simulations with Non-ideal MHD



Gressel+ 2015

Global Simulations with Non-ideal MHD

Suriano,... Suzuki, & Li 2019 ⇒
see also Riols & Lesur 2018



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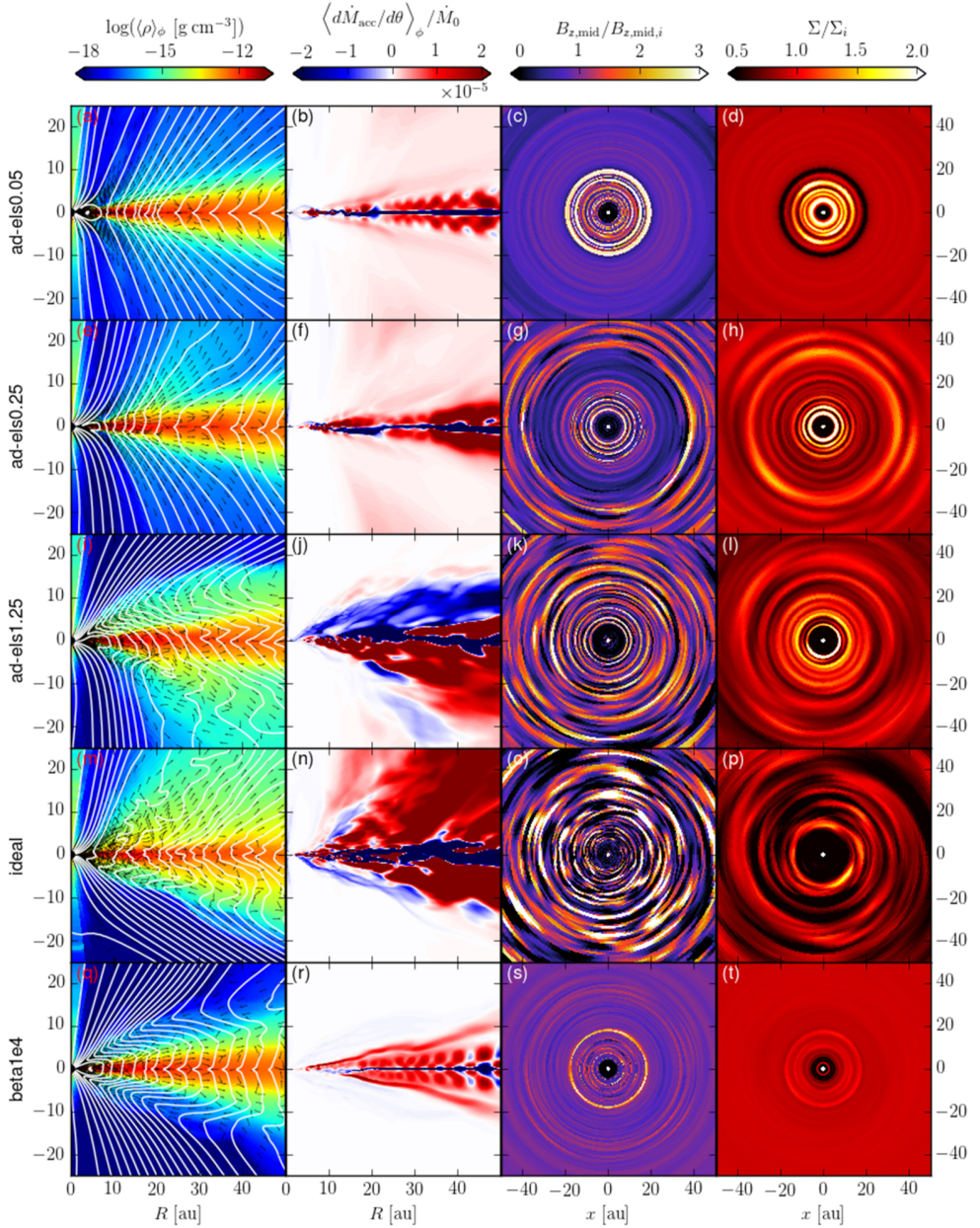

see also Flock+ 2015; Bai & Stone 2016;
Béthune, Lesur, & Ferreira 2017

Cartesian Shearing Box Simulations

Hawley et al. 1995;

Matsumoto & Tajima 1995; ...

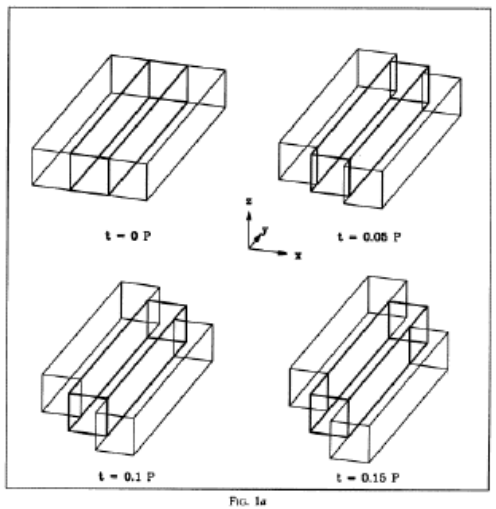

FIG. 1a

Magnetized Disk Winds in Local Simulations

Ideal MHD (Suzuki & Inutsuka 2009) Resistive MHD (Suzuki+ 2010)

Bai & Stone 2013; Lesur+ 2014; Fromang+ 2013; Simon+ 2018; Mori+ 2018; ...

Global vs.Local

Global vs.Local

Global Simulations

Global vs.Local

Global Simulations

- Handle global effects

Global vs.Local

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- But long-time simulations not easy
  - $\approx \mathbf{1000}$ rot. at $\boldsymbol{R}_{\mathbf{in}} \Rightarrow \lesssim$ 100 rot. at $\boldsymbol{R}_{\mathbf{mid}}$

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- Capture fine-scale structure/turbulence

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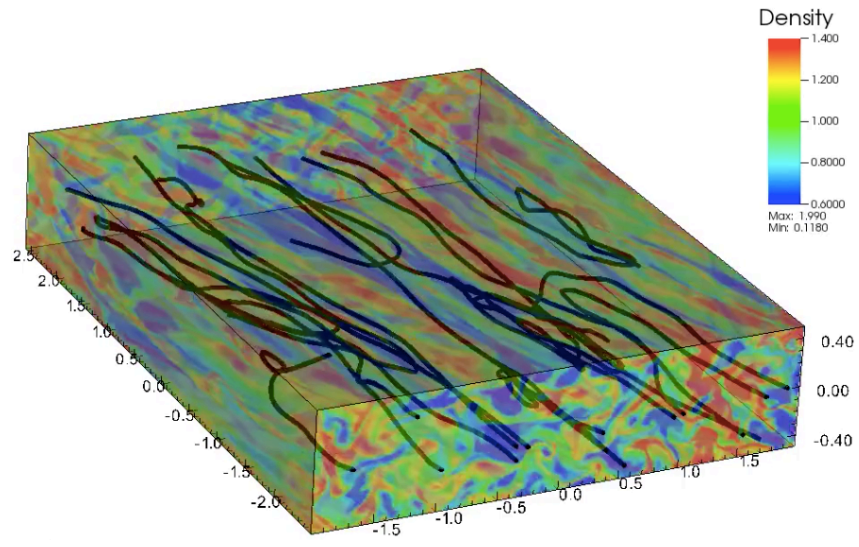
- But disadvantages ... $\Rightarrow$ Next Page

Cartesian Shearing Box

Some Disadvantages



Cartesian Shearing Box

Some Disadvantages

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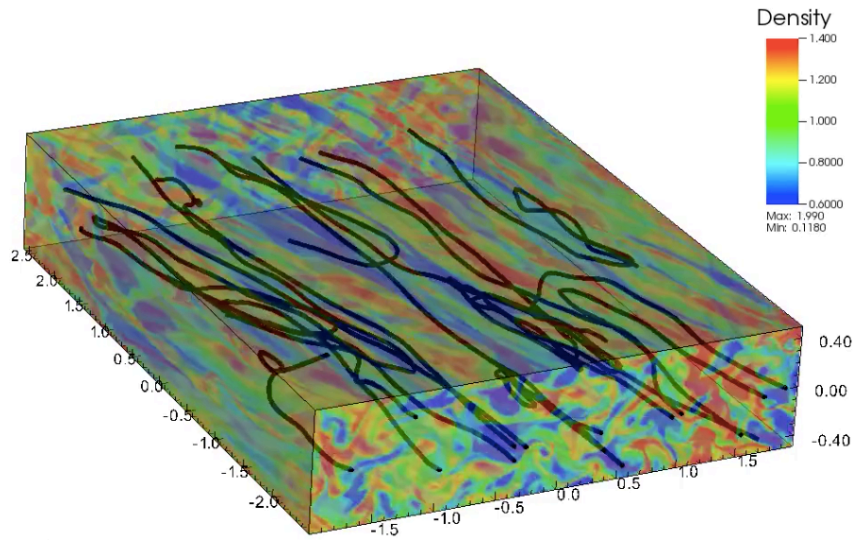
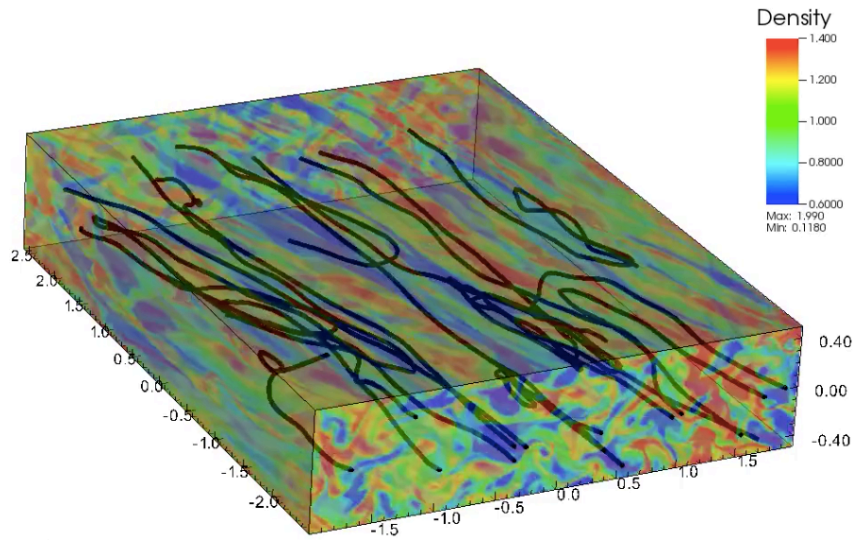


Cartesian Shearing Box

Some Disadvantages

- Neglect the Curvature
- $\pm x$ symmetry
  The central star located on either left or right



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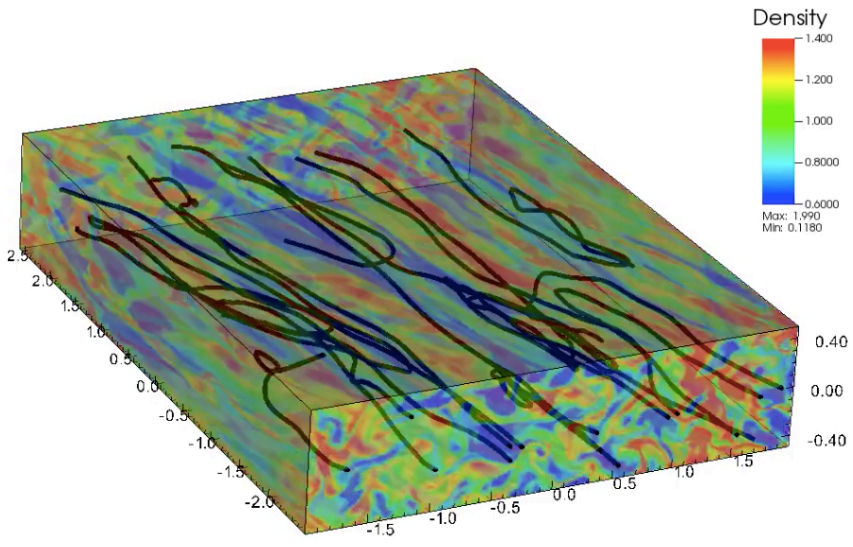

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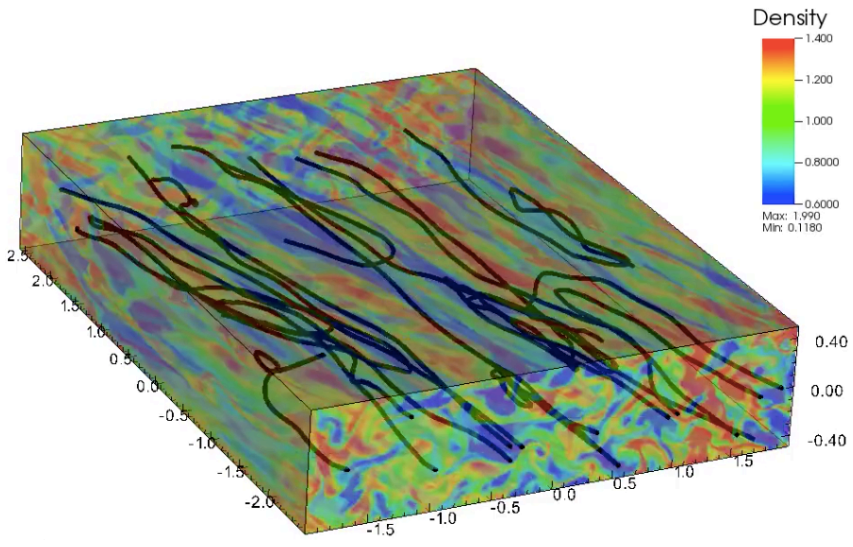

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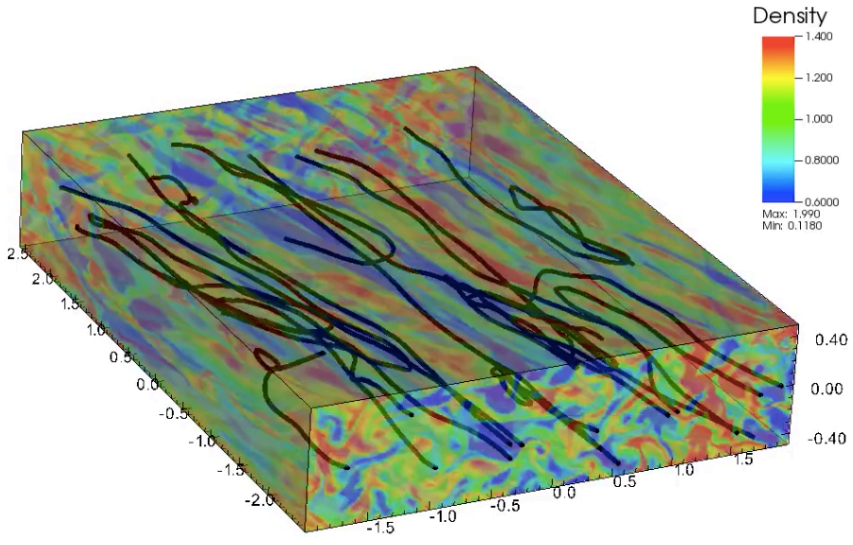
- No Net Gas Accretion
- The direction of angular momentum NOT defined
- Removal of Angular Momentum by Disk Winds NOT well-defined



Zoom-in & Zoom-out

Global ⇐ ⇒ Local

Zoom-in & Zoom-out

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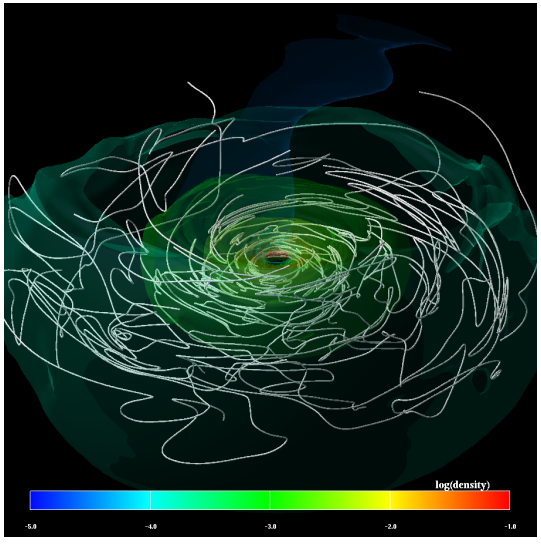

Spherical

(r, θ, ϕ)

Zoom-in & Zoom-out

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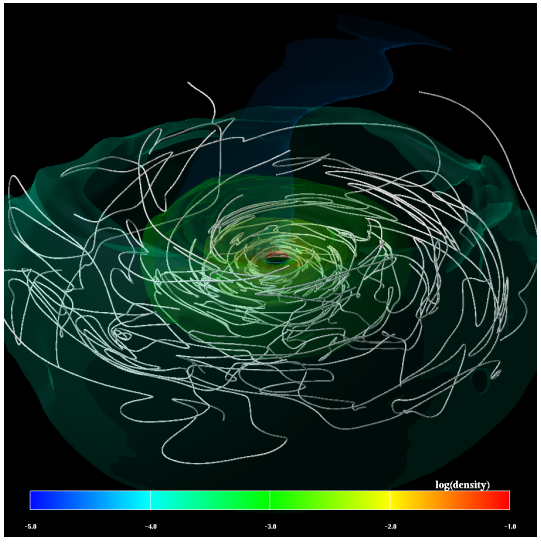

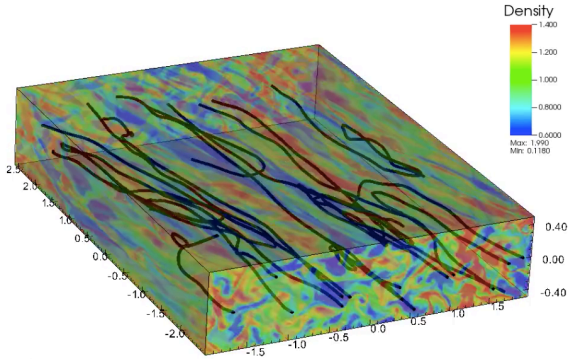

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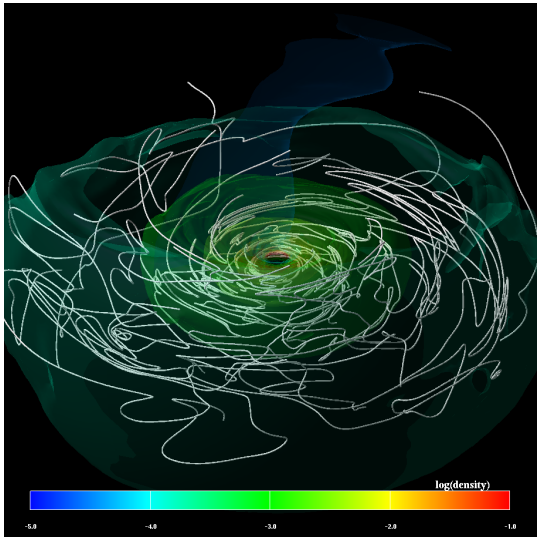

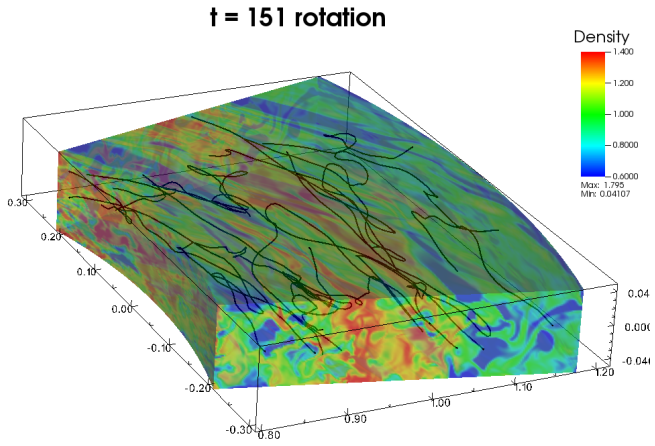

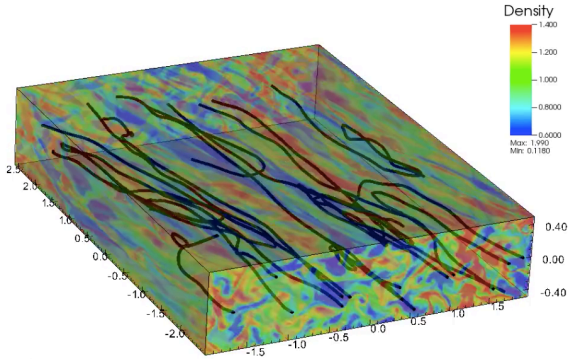

Spherical (r, θ, ϕ)

Cylindrical (r, ϕ, z)

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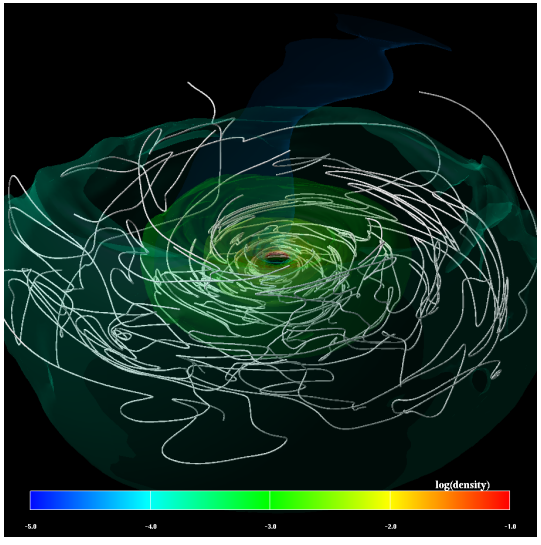
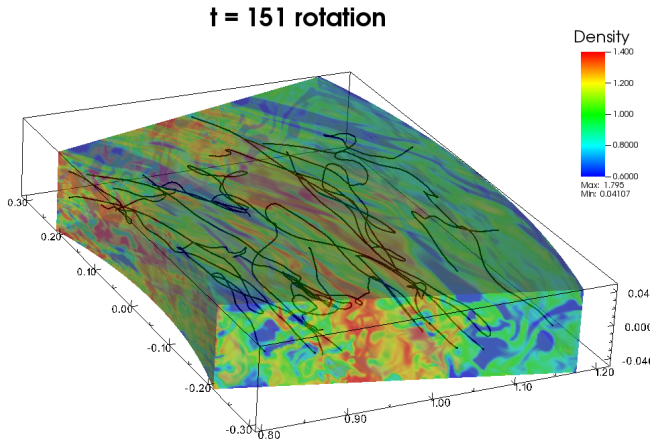
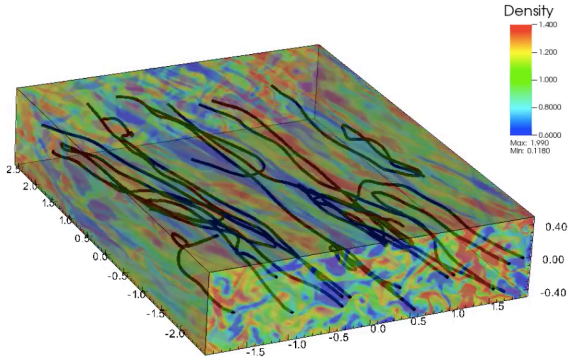
A New Approach: "Cylindrical Shearing Box"

Zoom-in & Zoom-out

Global ⇐ ⇒ Local







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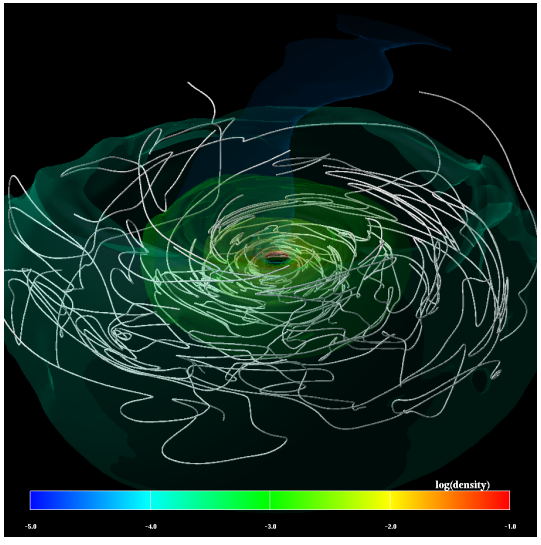

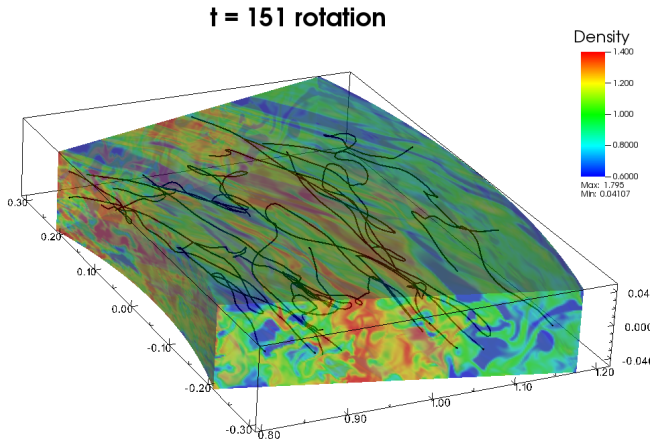

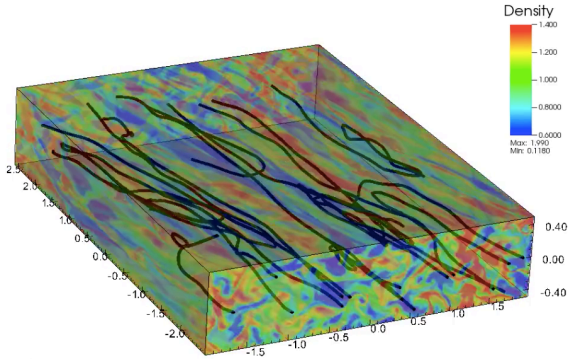

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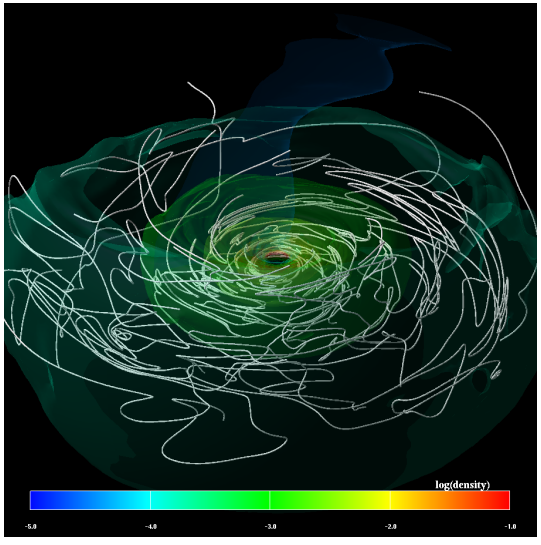

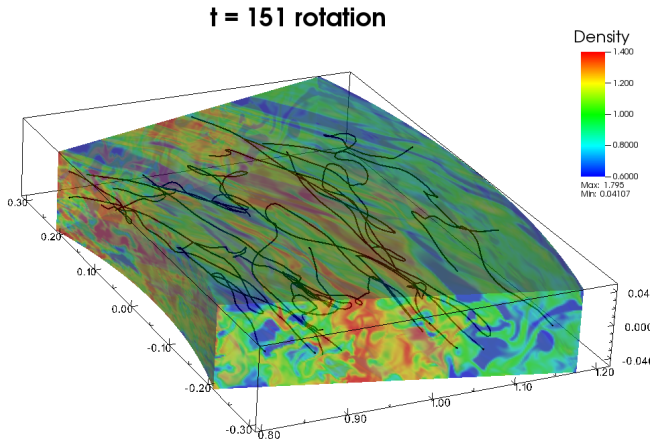

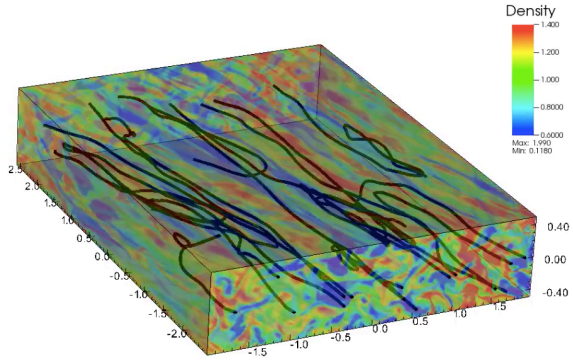

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⇒ can handle the net accretion ?

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  Add curvature terms (Brandenburg+ 1996)

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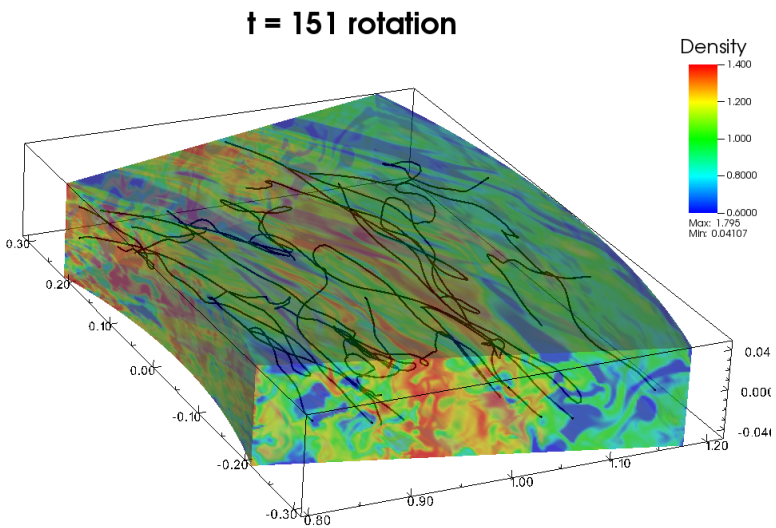
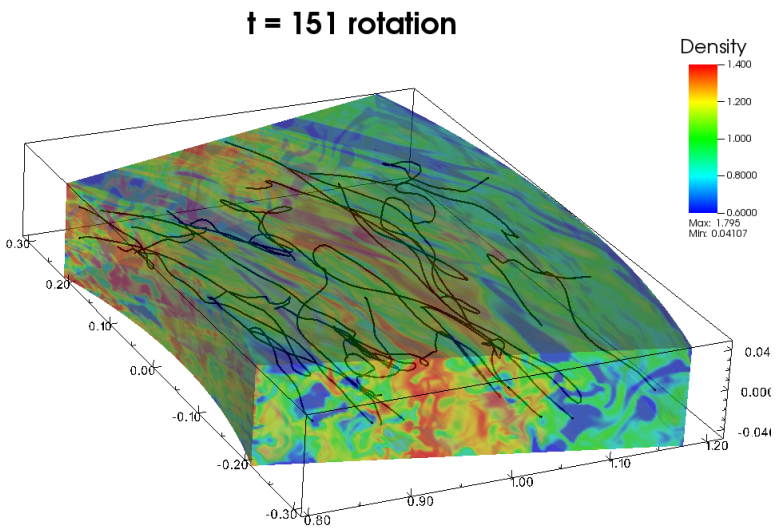
Unphysical oscillations excited
⇒ Damping zone treatment

Cylindrical Shearing Box



Cylindrical Shearing Box

Key : Boundary Condition at $R_{\pm}$

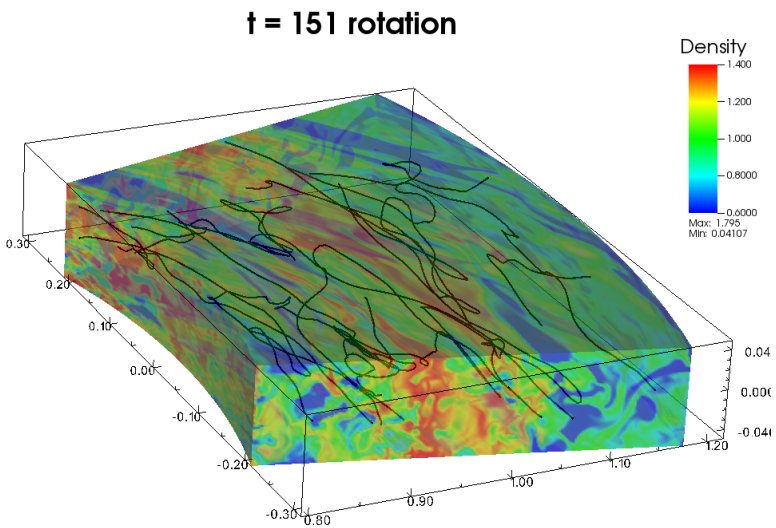


Cylindrical Shearing Box

Key : Boundary Condition at $R_{\pm}$

- Shear: $A(R_{\pm}, \phi, z) = A(R_{\mp}, \phi \pm \Delta\Omega_{eq} t, z)$

where $\Delta\Omega_{eq} = \Omega_{eq,-} - \Omega_{eq,+}$



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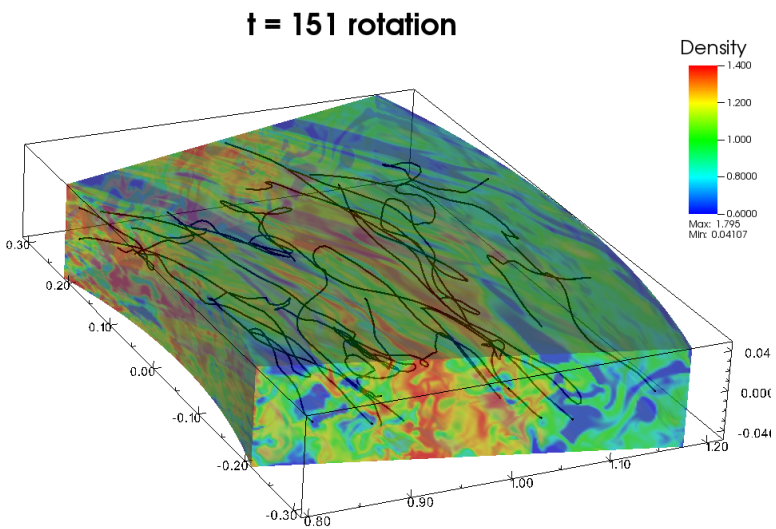
- Radial Boundary Condition

  $\Leftarrow$ Conservation Laws

  of Mass+Momentum+(Energy)+B

Conserved quantities, A, at R_- & R_+



$$A = \begin{cases} \rho v_R R \\ \rho v_R^2 R \\ (\rho v_R v_\phi + B_\phi B_R/4\pi)/\Omega_{\rm eq} \\ \rho v_R v_z R \\ v_R B_\phi - v_\phi B_R \\ (v_z B_R - v_R B_z) R \\ \text{Energy} \end{cases}$$

Shearing Radial Boundary

- Mass:
  $\partial_t \rho + R^{-1}\partial_R(\rho v_R R) + \partial_\phi(\cdots) + \partial_z(\cdots) = 0$
- Momentum– R:
  $\partial_t(\rho v_R) + R^{-1}\partial_R(\rho v_R^2 R) + \cdots = 0$
- Momentum– ϕ (Angular Momentum):
  $\partial_t(\rho v_\phi R) + \partial_R[(\rho v_R v_\phi + B_R B_\phi/4\pi)R^2] + \cdots = 0$
- Momentum– z:
  $\partial_t(\rho v_z) + R^{-1}\partial_R(\rho v_R v_z R) + \cdots = 0$
- Induction eq.– ϕ
  $\partial_t B_\phi = \partial_z(\cdots) - \partial_R(v_R B_\phi - v_\phi B_R)$
- Induction eq.– z
  $\partial_t B_z = R^{-1}\partial_R[(v_z B_R - v_R B_z)R] - \partial_\phi(\cdots)$
- Energy Equation

Simulation Details

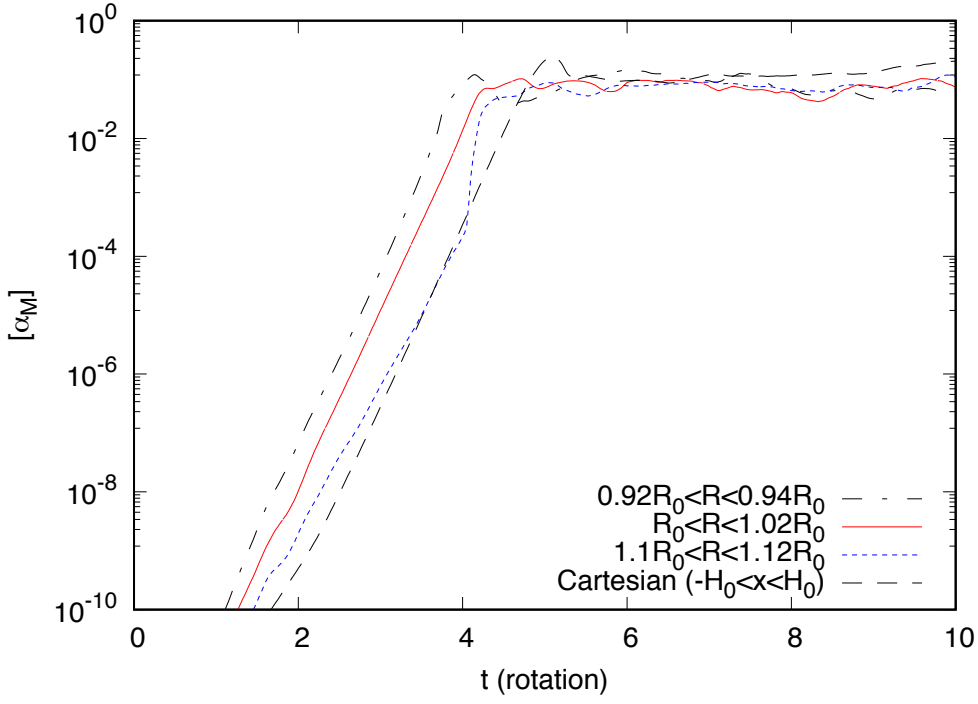
- z–Unstratified
- Ideal MHD
- $c_{s,0}/R_0\Omega_{\mathrm{Kep},0} = H_0/R_0 = 0.1$
- local isothermal: $T = T_0\left(\frac{R}{R_0}\right)^{-1}$
- Initial $B_z = B_{z,0}\left(\frac{R}{R_0}\right)^{-1}$; $\rho = \rho_0\left(\frac{R}{R_0}\right)^{-1}$
  with $\beta_{z,0} = 8\pi p/B^2_{z,0} = 10^3$
- Box Size $(L_R, L_\phi, L_z) = (4H_0, (5\pi/3)H_0, H_0)$

Cylindrical Shearing Box (CySB)

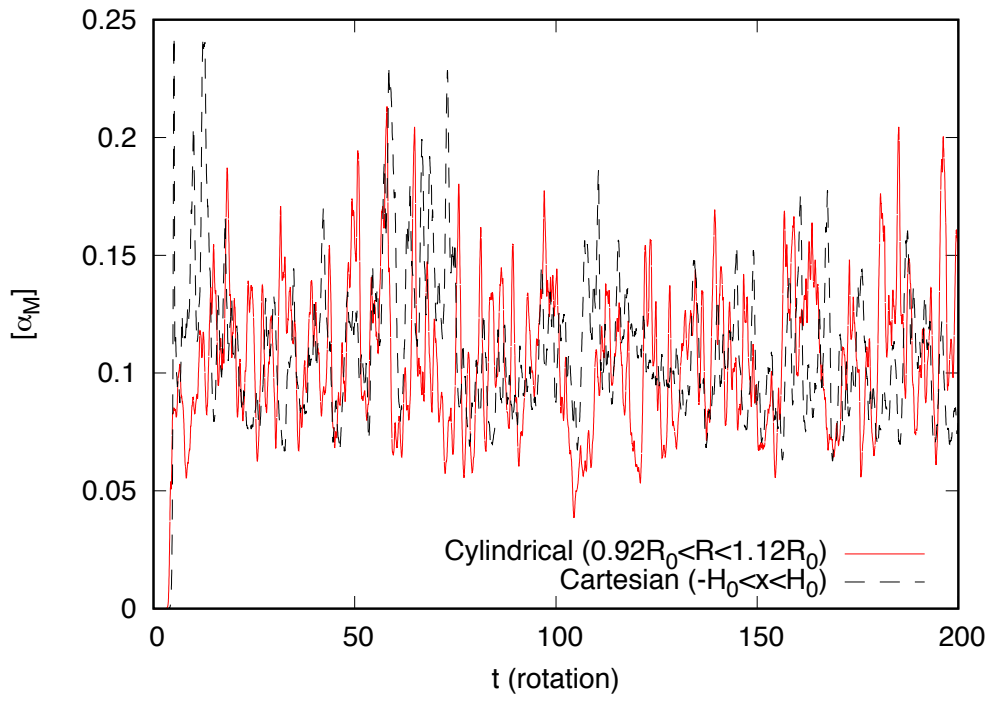
Time Evolution of α_{M}

$$\alpha_{\mathrm{M}} = B_R B_\phi / 4\pi p$$

$t = 0 - 10$ rotations



$t = 190 - 200$ rotations

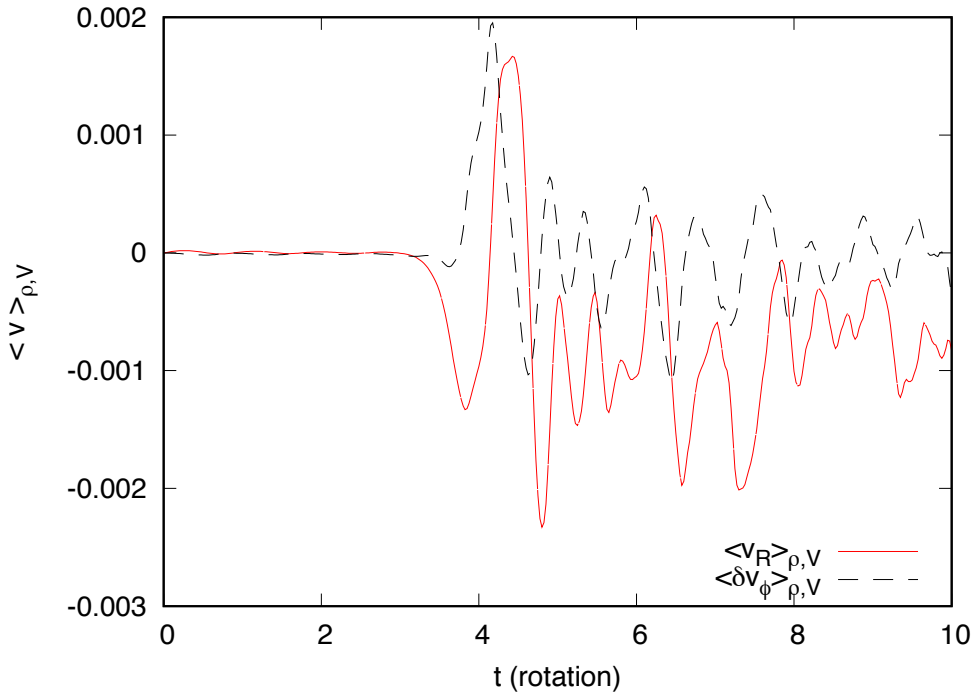


Cartesian (black dashed line)

vs. Cylindrical (red solid & other lines)

Mass Accretion

$t = 0 - 10$ rotations



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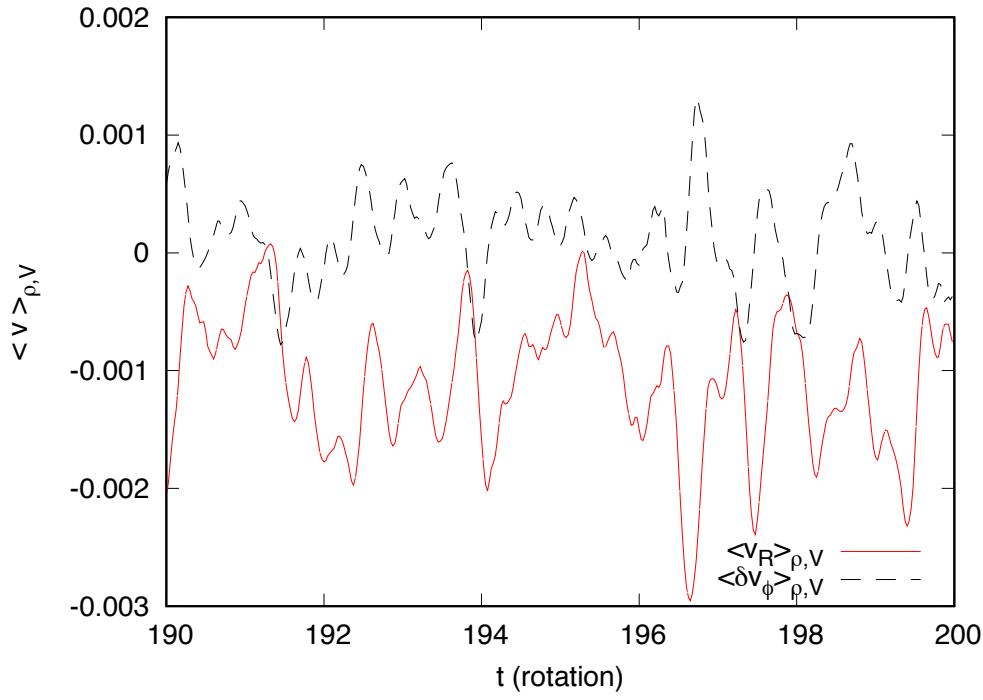

Red: v_R, Black: δv_ϕ

Mass Accretion

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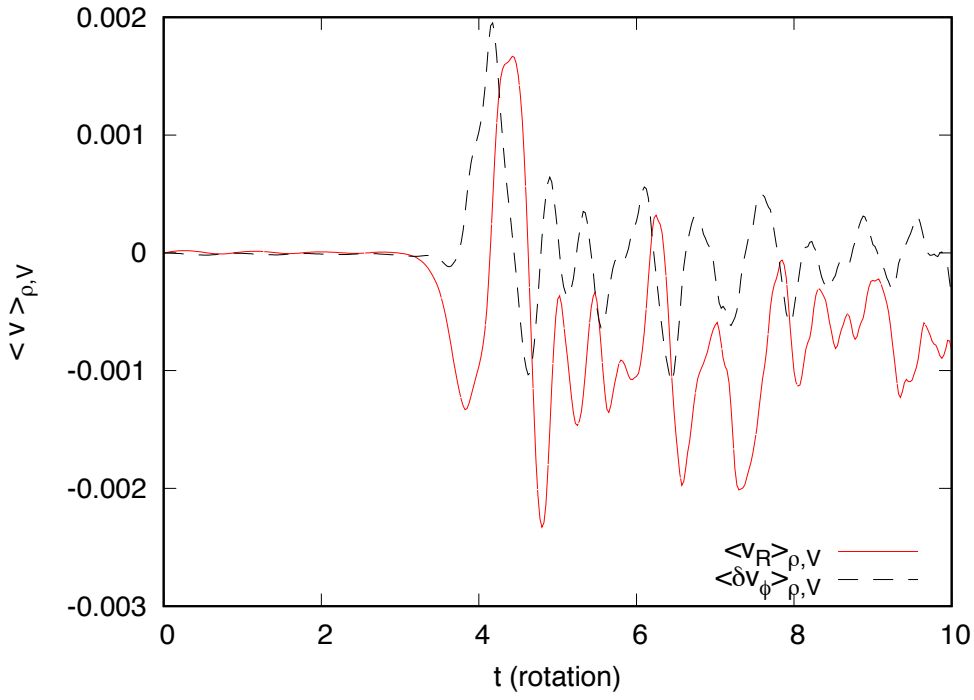

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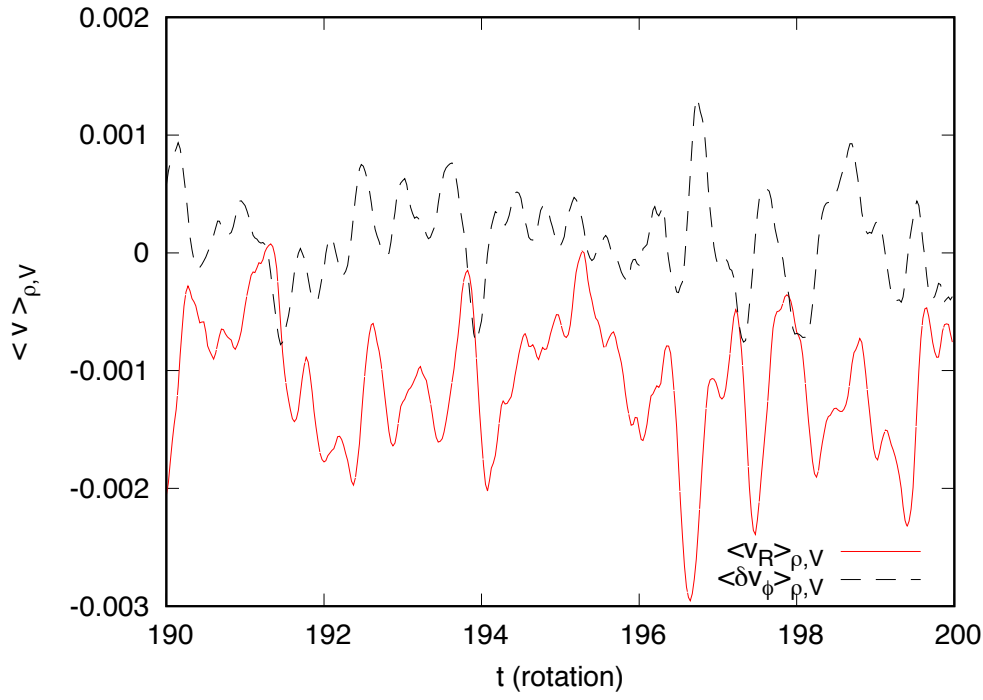


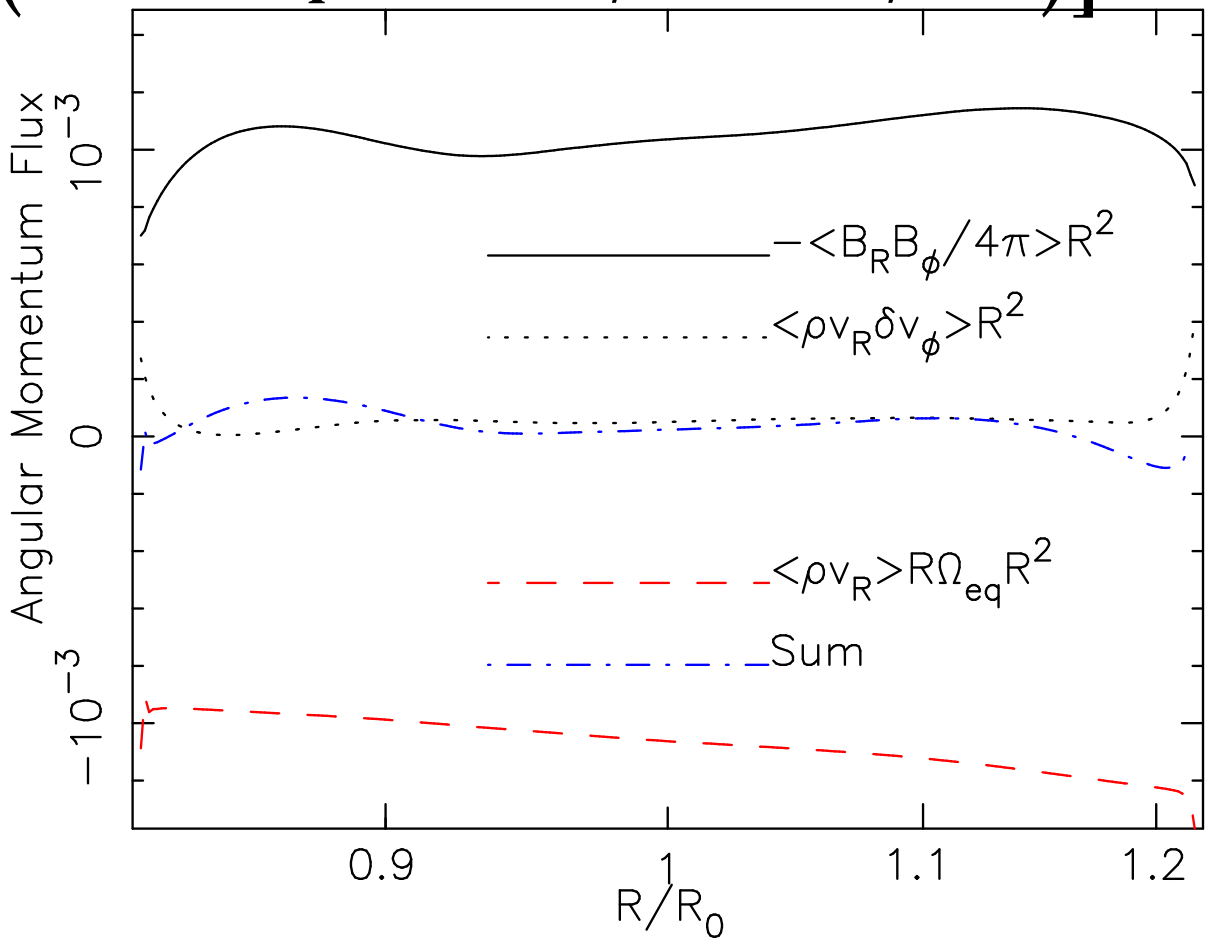
Red: v_R, Black: δv_ϕ

Inward Mass Accretion is naturally induced.

Angular Momentum Flows

Under the steady-state condition:

$$\frac{\partial}{\partial R}\left[R^2\left(\rho v_R R\Omega_{\rm eq} + \rho v_R \delta v_\phi - B_R B_\phi/4\pi\right)\right] = 0$$



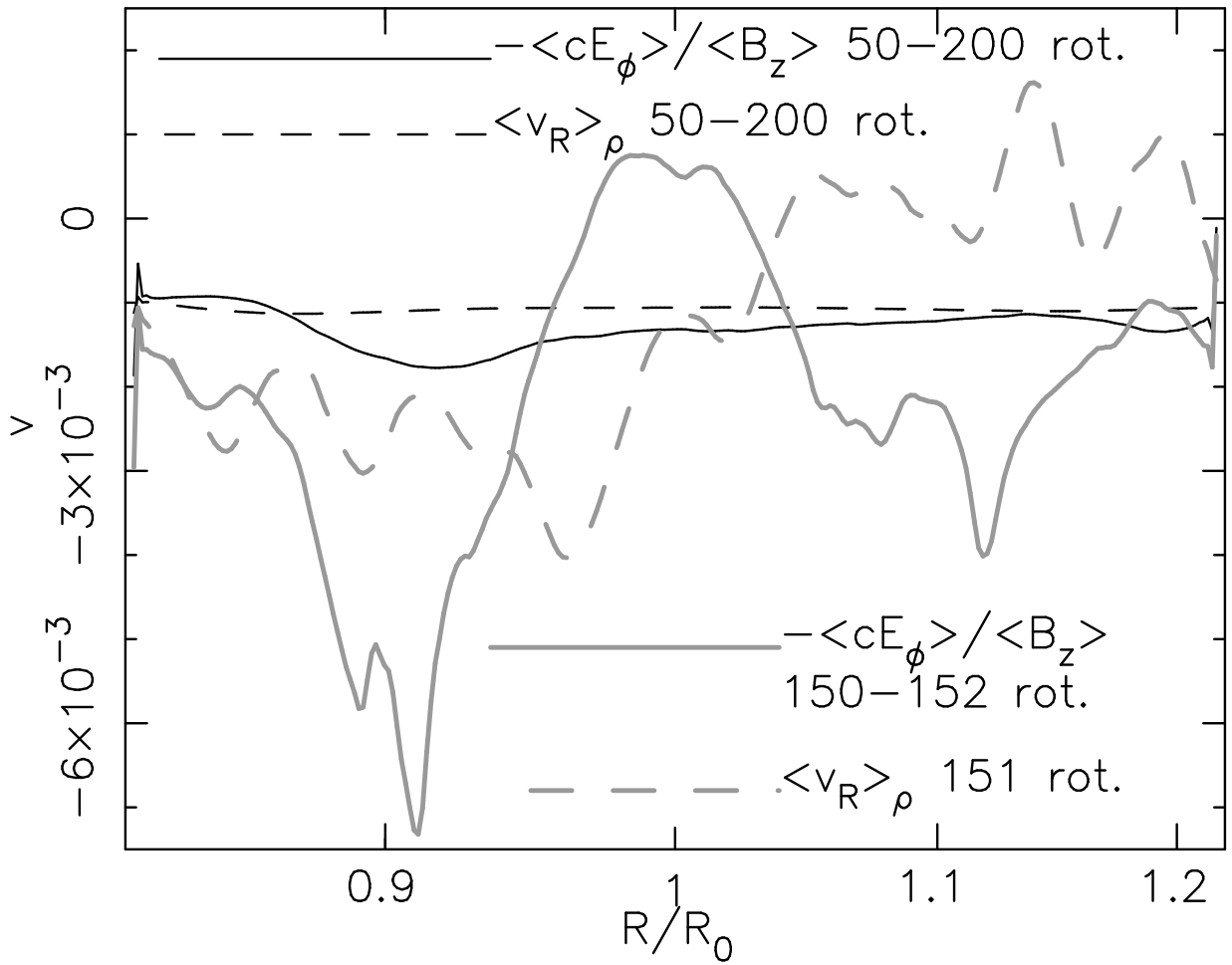
Outward A.M. flux (turbulence; black lines)

+ Inward A.M. flux (accretion; red line) ≈ 0

Accretion & B_z Advection

$$\frac{\partial}{\partial t}(RB_z) = \frac{\partial}{\partial R}\left[R\left(v_z B_R - v_R B_z\right)\right]$$

$$\Rightarrow \langle v_{R,B_z}\rangle = \langle R(v_R B_z - v_z B_R)\rangle / \langle RB_z\rangle = \langle cE_\phi\rangle / \langle B_z\rangle$$



$\boldsymbol{B}$ field and gas are slipped via turbulent diffusion

Limitations / Open Issues

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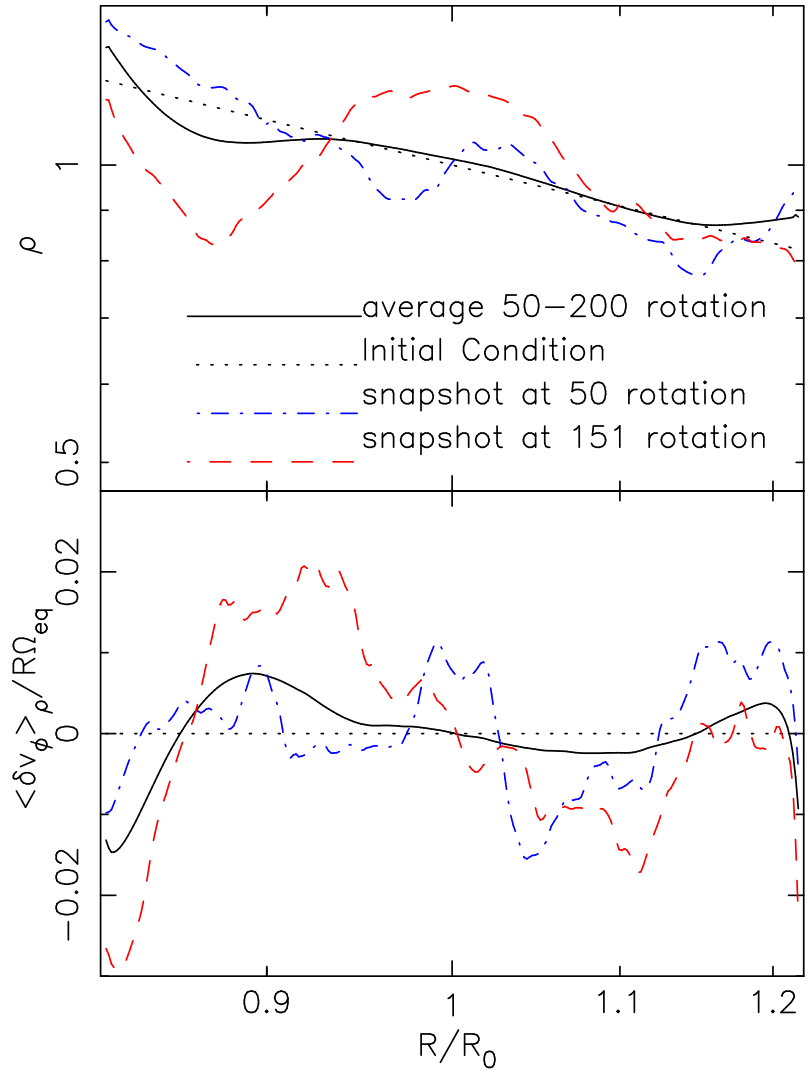
  - A zonal flows via boundary effects ?



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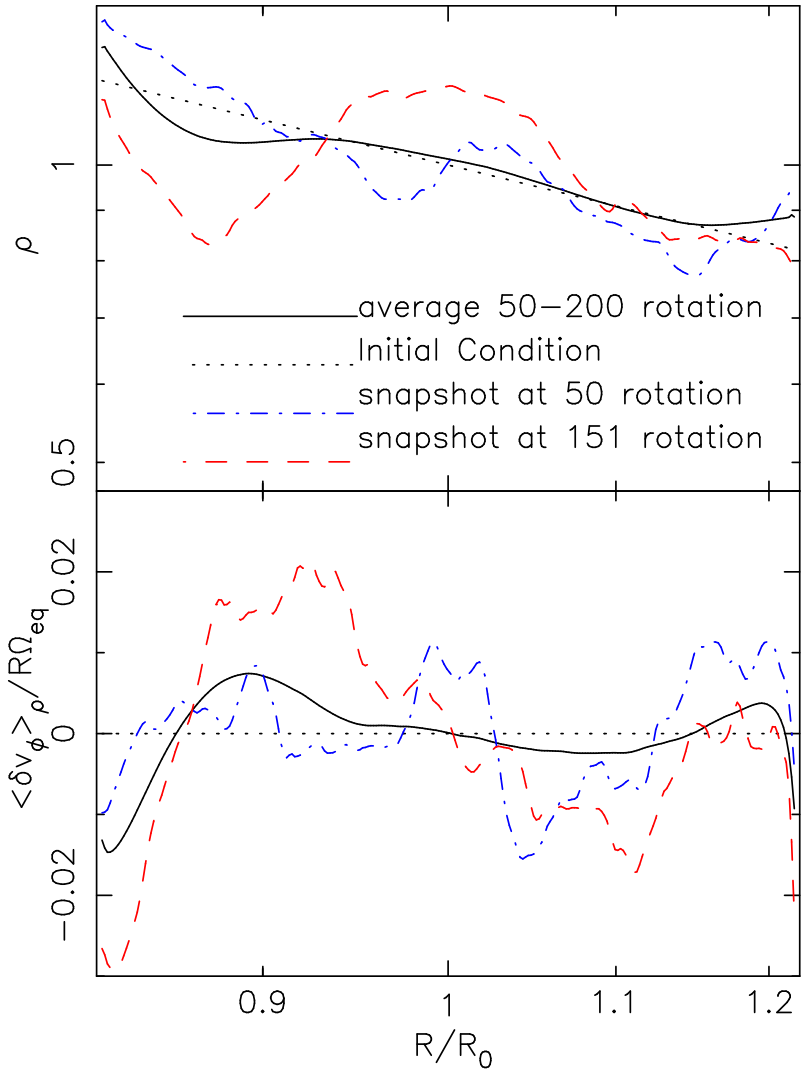

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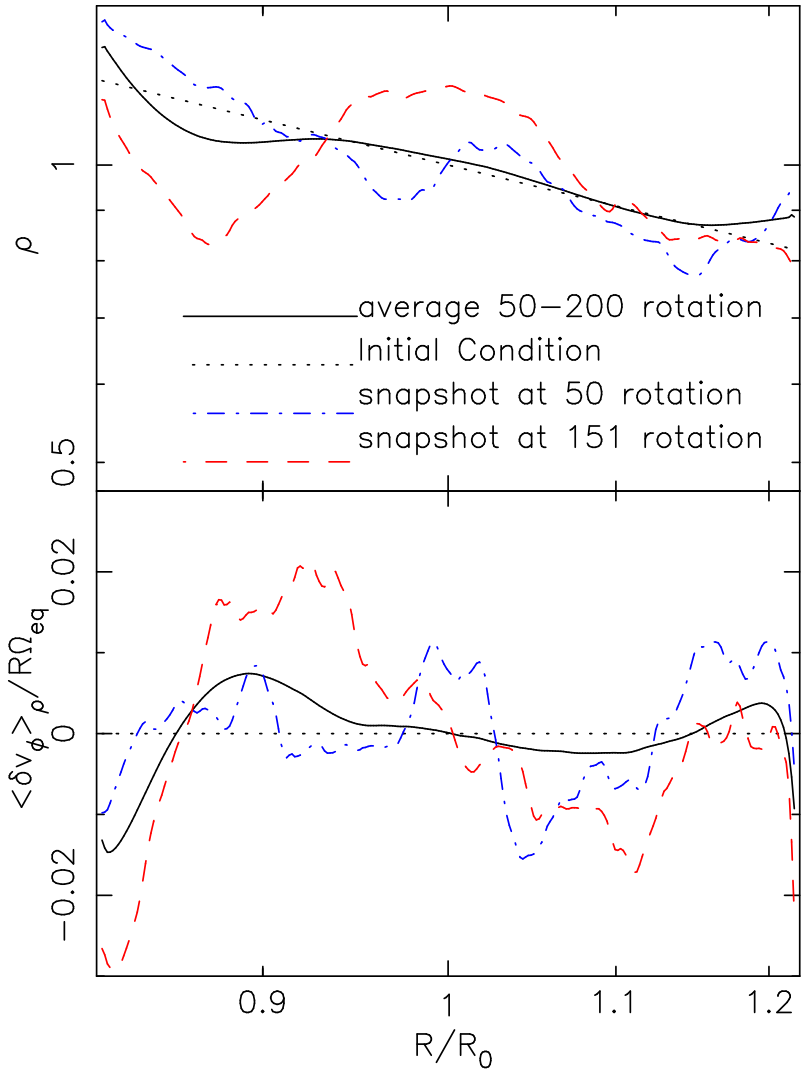

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–still on-going

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Need Your Help;

Comments & Criticism are all welcome.