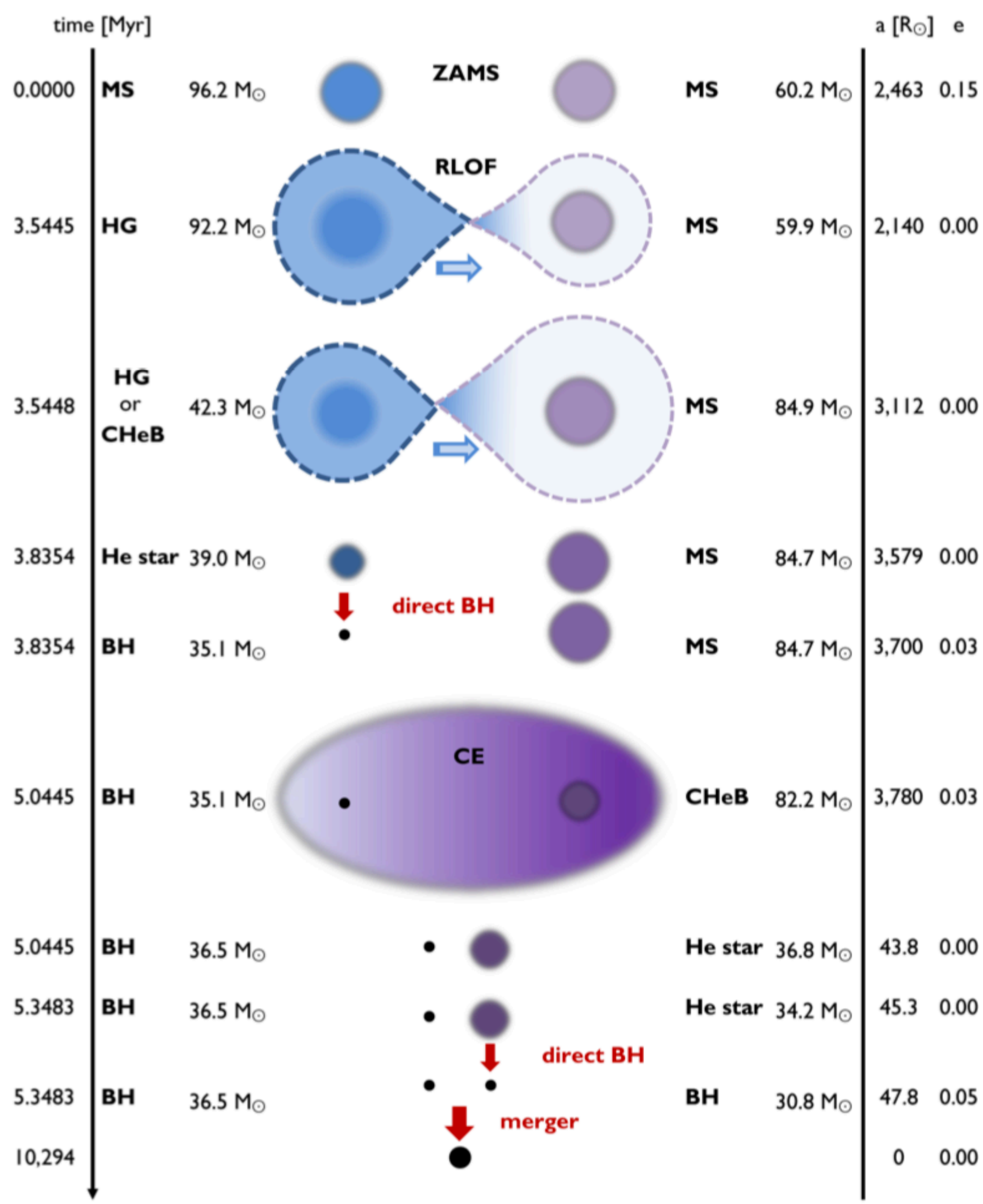
Plan for binary population synthesis of extreme metal-poor stars

谷川 衝 (東京大学)
Ataru Tanikawa (The University of Tokyo)

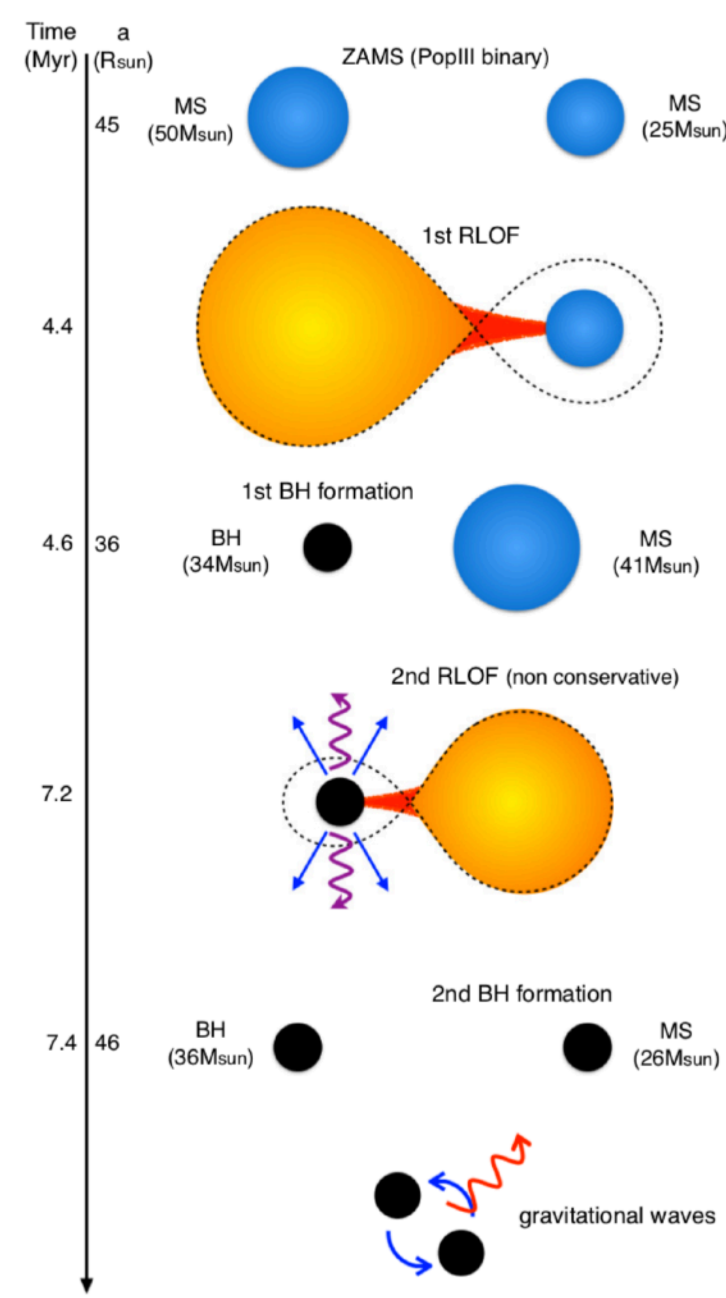
Group-A Boot Camp
at Yousen-kaku Iwamatsu ryokan

Binary population synthesis

Belczynski et al. (2016)



Inayoshi et al. (2017)



Overview of the plan

- Make fitting formula (FF) of evolution of extreme metal-poor (EMP) stars
- Study binary population synthesis of EMP stars

FFs of previous studies

- $Z/Z_\odot=0$ (Kinugawa, Inayoshi, Hotokezaka, Nakauchi, Nakamura 2014, MNRAS, 442, 2963)
- $Z/Z_\odot=0.005$ - 1.5 (Hurley, Pols, Tout 2000, MNRAS, 315, 543)

Kinugawa's FF

$$\log(R_{\rm H}/{\rm R}_\odot) = \log(R_{\rm ZAMS}/{\rm R}_\odot) + a_{\rm H}\tau_{\rm H} + b_{\rm H}\tau_{\rm H}^{10} + c_{\rm H}\tau_{\rm H}^{500} + d_{\rm H}\tau_{\rm H}^{3}, \quad (4)$$

$$(R_{\rm ZAMS}/{\rm R}_\odot) = 1.22095 + 2.70041\times10^{-2}(M/10\ {\rm M}_\odot) + 0.135427(M/10\ {\rm M}_\odot)^2 - 1.95541\times10^{-2}(M/10\ {\rm M}_\odot)^3 + 8.7585\times10^{-4}(M/10\ {\rm M}_\odot)^4, \quad (1)$$

$$(R_{\rm H}^{\rm e}/{\rm R}_\odot) = 0.581309 + 2.27745(M/10\ {\rm M}_\odot) + 6.63321\times10^{-3}(M/10\ {\rm M}_\odot)^3, \quad (2)$$

$$(t_{\rm H}/{\rm Myr}) = 1.78652 + 10.4323(M/10\ {\rm M}_\odot)^{-1} + 3.70946(M/10\ {\rm M}_\odot)^{-2} + 2.04264(M/10\ {\rm M}_\odot)^{-3}, \quad (3)$$

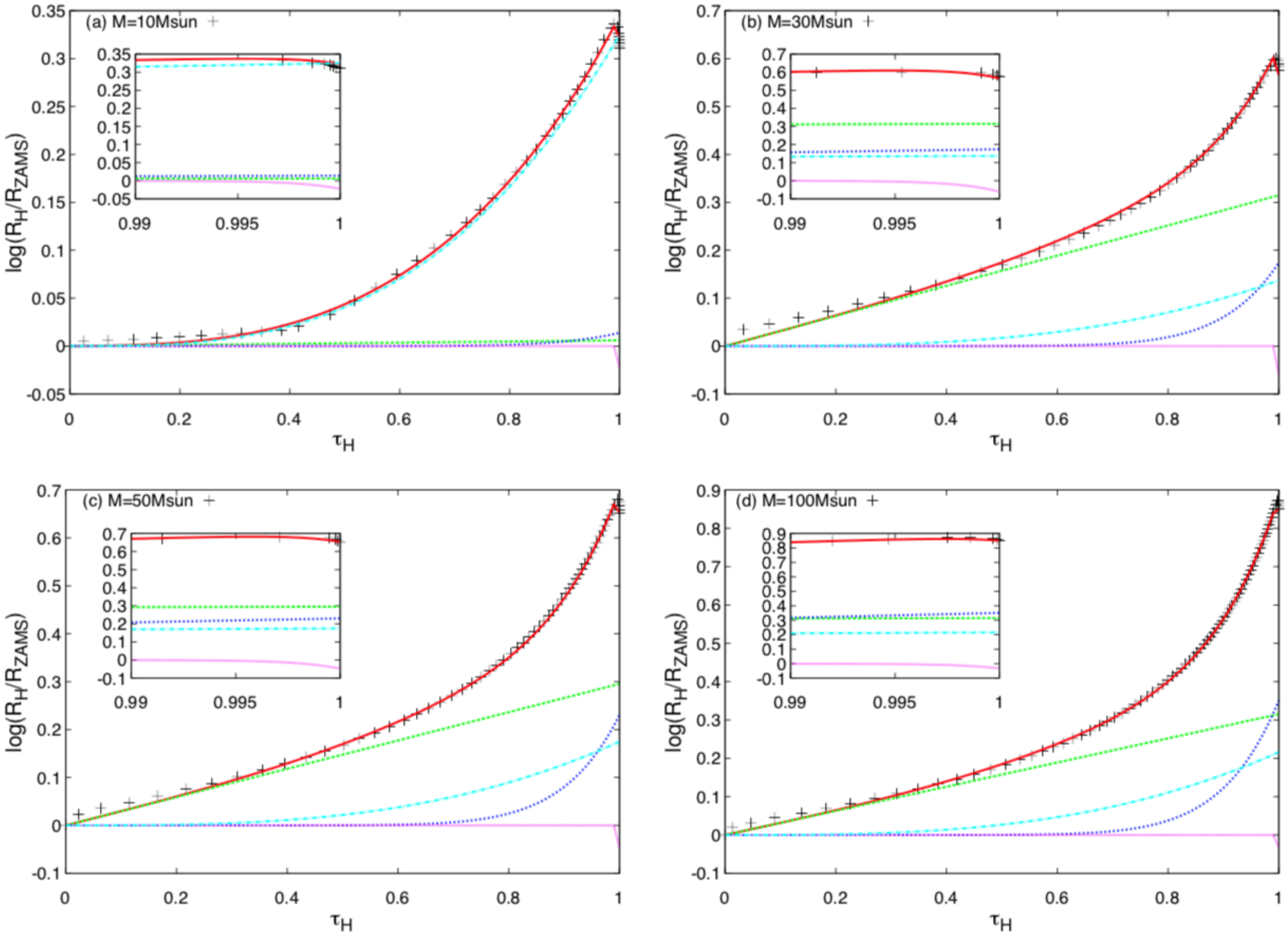
$$a_{\rm H} = \begin{cases} -0.430873 + 0.520408(M/10\ {\rm M}_\odot) \\ -7.99762\times10^{-2}(M/10\ {\rm M}_\odot)^2 \\ -3.55095\times10^{-3}(M/10\ {\rm M}_\odot)^3 \\ \qquad (10\ {\rm M}_\odot \leqslant M < 30\ {\rm M}_\odot), \\ 0.476498 - 9.07537\times10^{-2}(M/10\ {\rm M}_\odot) \\ +1.43538\times10^{-2}(M/10\ {\rm M}_\odot)^2 \\ -6.89108\times10^{-4}(M/10\ {\rm M}_\odot)^3 \\ \qquad (30\ {\rm M}_\odot \leqslant M \leqslant 100\ {\rm M}_\odot), \end{cases} \quad (5)$$

$$b_{\rm H} = \begin{cases} 0.669345 - 1.5518(M/10\ {\rm M}_\odot) + 1.15116(M/10\ {\rm M}_\odot)^2 \\ -0.254811(M/10\ {\rm M}_\odot)^3 \\ \qquad (10\ {\rm M}_\odot \leqslant M < 20\ {\rm M}_\odot), \\ 3.02801\times10^{-2} + 6.48197\times10^{-2}(M/10\ {\rm M}_\odot) \\ -6.64582\times10^{-3}(M/10\ {\rm M}_\odot)^2 \\ +3.37205\times10^{-4}(M/10\ {\rm M}_\odot)^3 \\ \qquad (20\ {\rm M}_\odot \leqslant M \leqslant 100\ {\rm M}_\odot), \end{cases} \quad (6)$$

$$c_{\rm H} = \begin{cases} 5.63328\times10^{-2} - 9.88927\times10^{-2}(M/10\ {\rm M}_\odot) \\ +2.00071\times10^{-2}(M/10\ {\rm M}_\odot)^2 \\ \qquad (10\ {\rm M}_\odot \leqslant M < 30\ {\rm M}_\odot), \\ -0.128025 + 3.63928\times10^{-2}(M/10\ {\rm M}_\odot) \\ -5.43719\times10^{-3}(M/10\ {\rm M}_\odot)^2 \\ +2.75137\times10^{-4}(M/10\ {\rm M}_\odot)^3 \\ \qquad (30\ {\rm M}_\odot \leqslant M \leqslant 100\ {\rm M}_\odot), \end{cases} \quad (7)$$

$$d_{\rm H} = \log(R_{\rm H}^{\rm e}/R_{\rm ZAMS}) - a_{\rm H} - b_{\rm H} - c_{\rm H}. \quad (8)$$

Kinugawa's FF



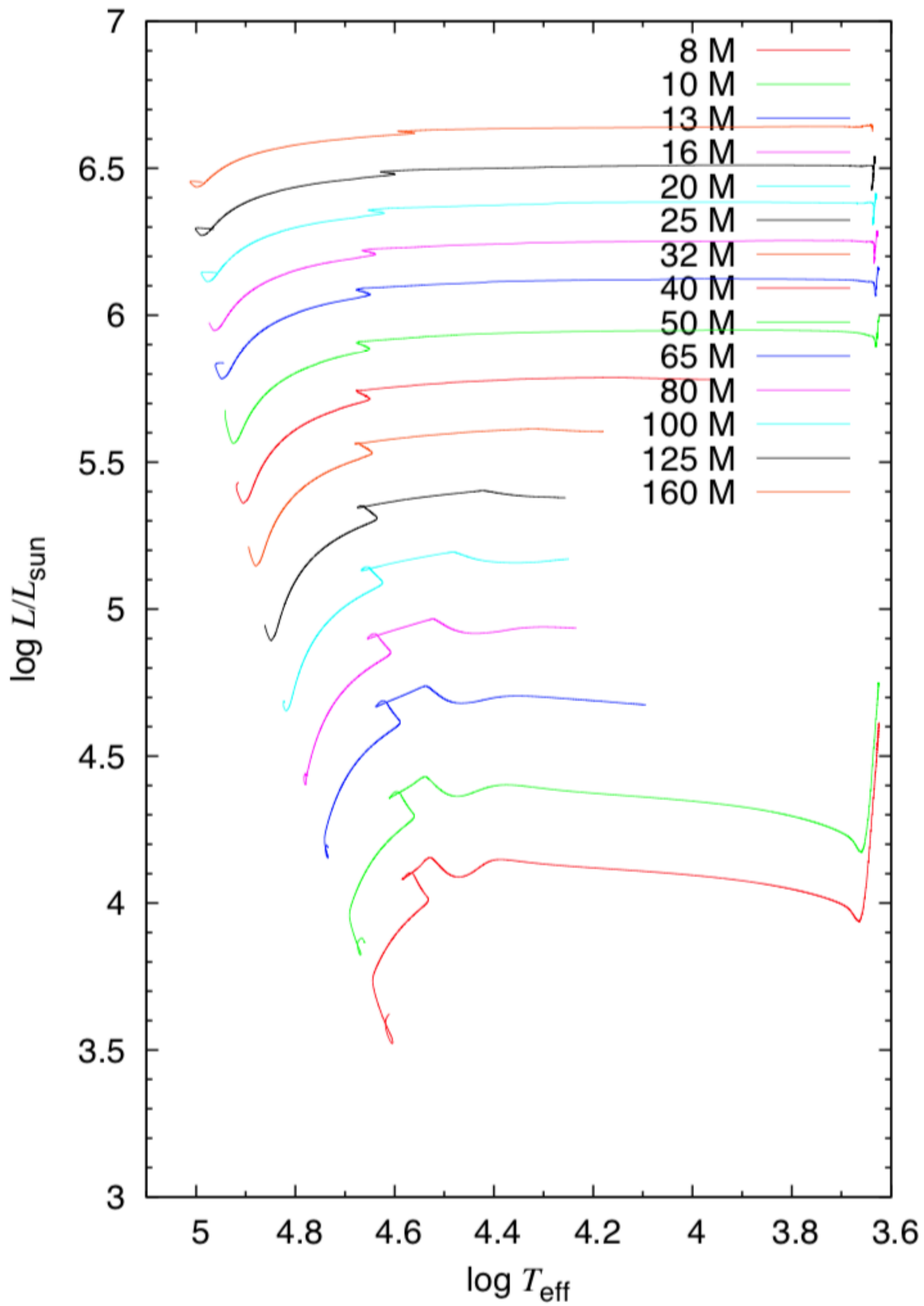
Our FF

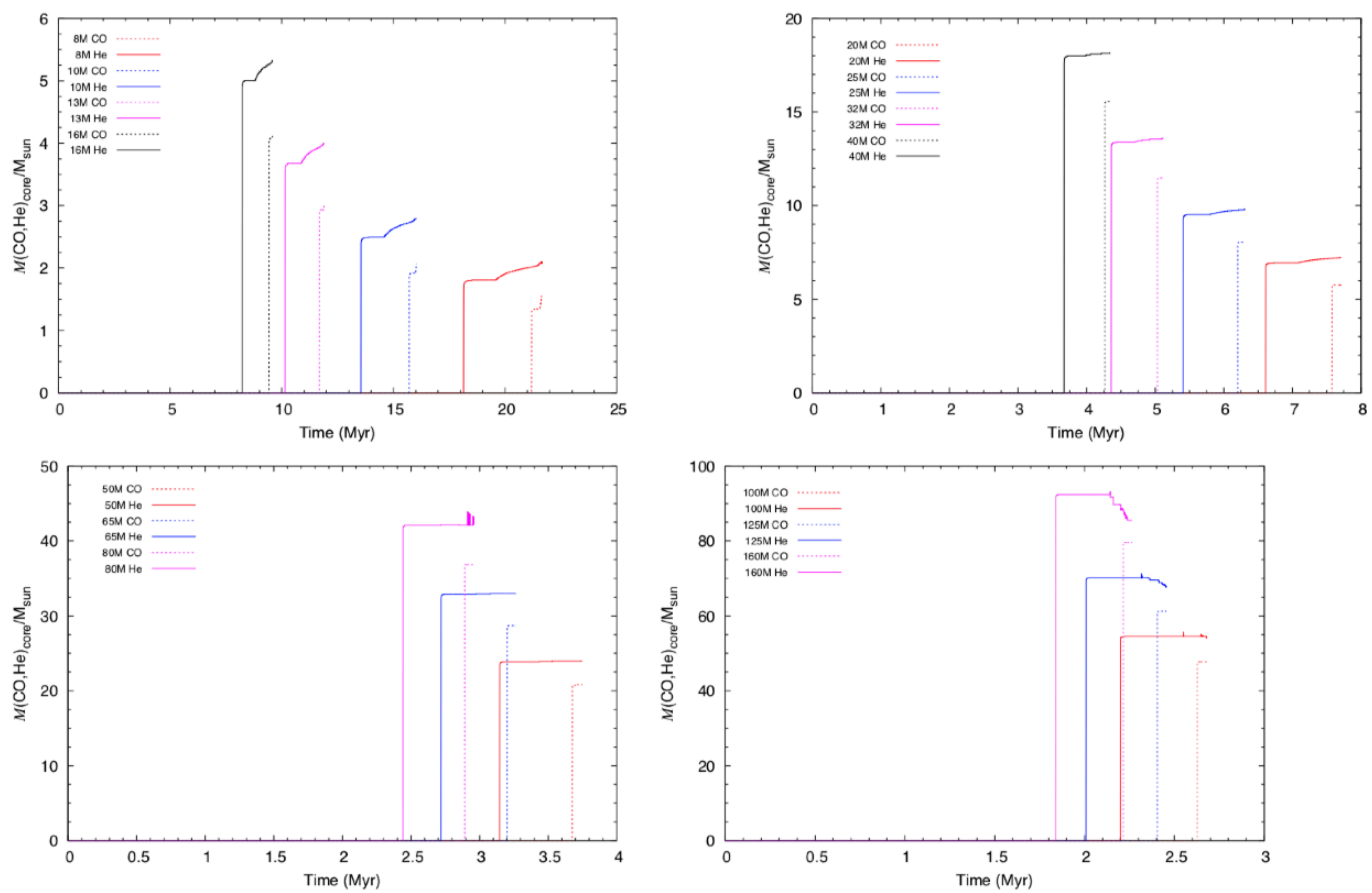
- Make FFs of $Z/Z_{\odot}=10^{-8}$, 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, similarly to Kinugawa's FFs
- Interpolate FFs as a function of Z
    - But it is unclear the interpolation is possible in the range of 5 orders of magnitude.
    - Note that Hurley's interpolation range is less than 3 orders of magnitude.

Treatment of stellar wind

- The FFs do not consider stellar wind.
- When we follow the evolution of a star with stellar wind, the star moves from a evolution track to another evolution track.

Data of $Z/Z_{\odot}=10^{-8}$ by Takashi Yoshida

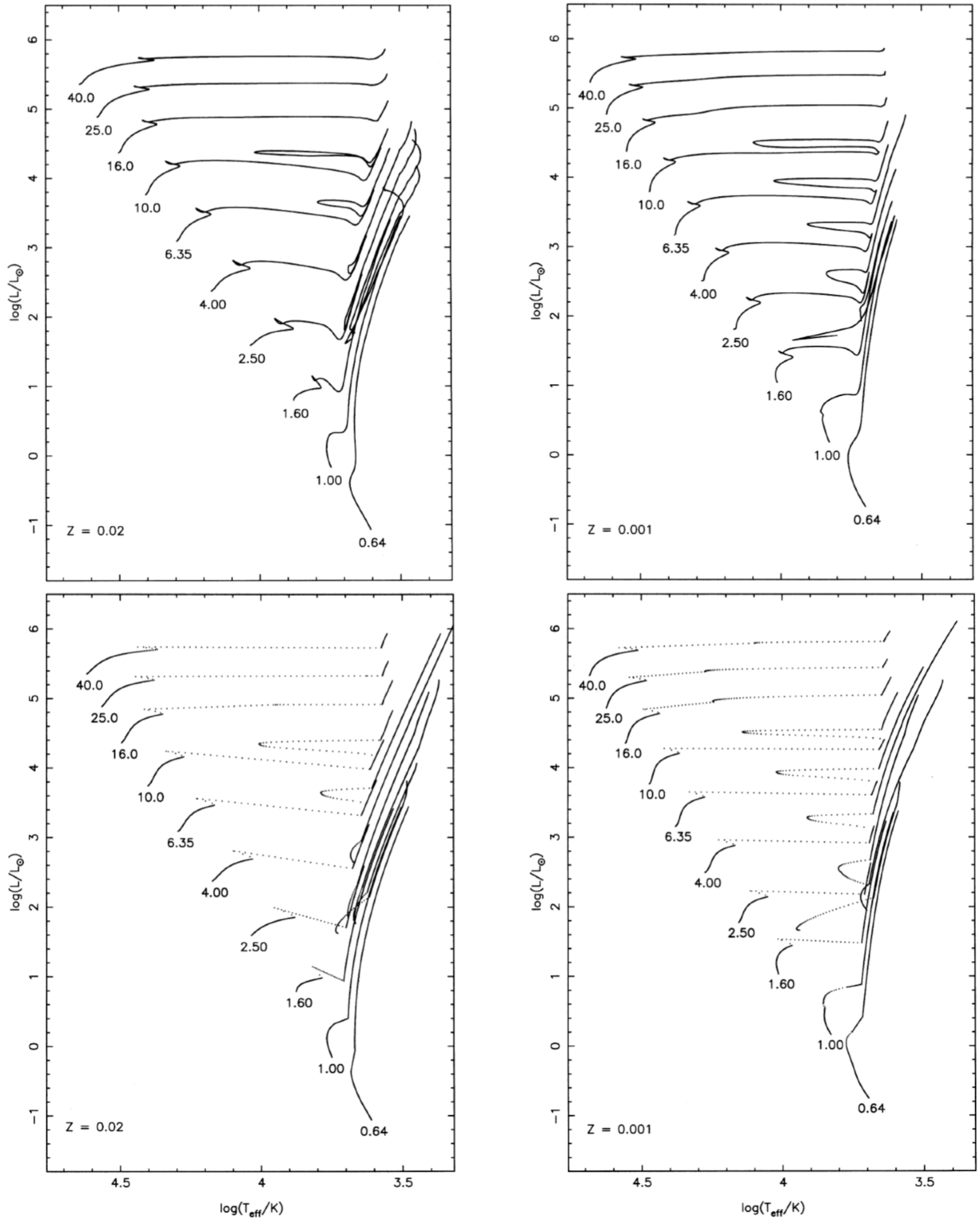




Hurley’s FF

The results of a stellar evolution code (Pols et al. 1998)

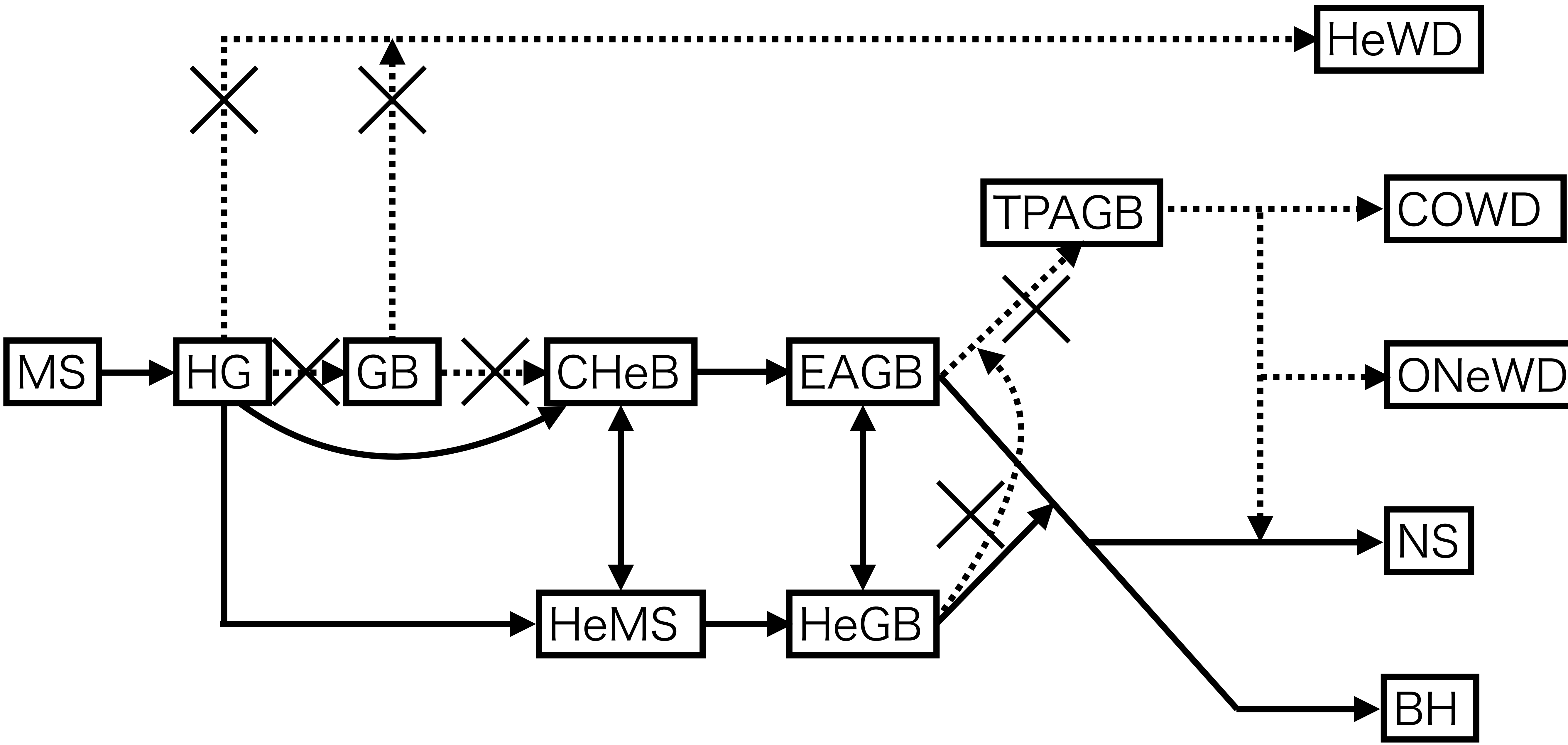
Hurley’s FF



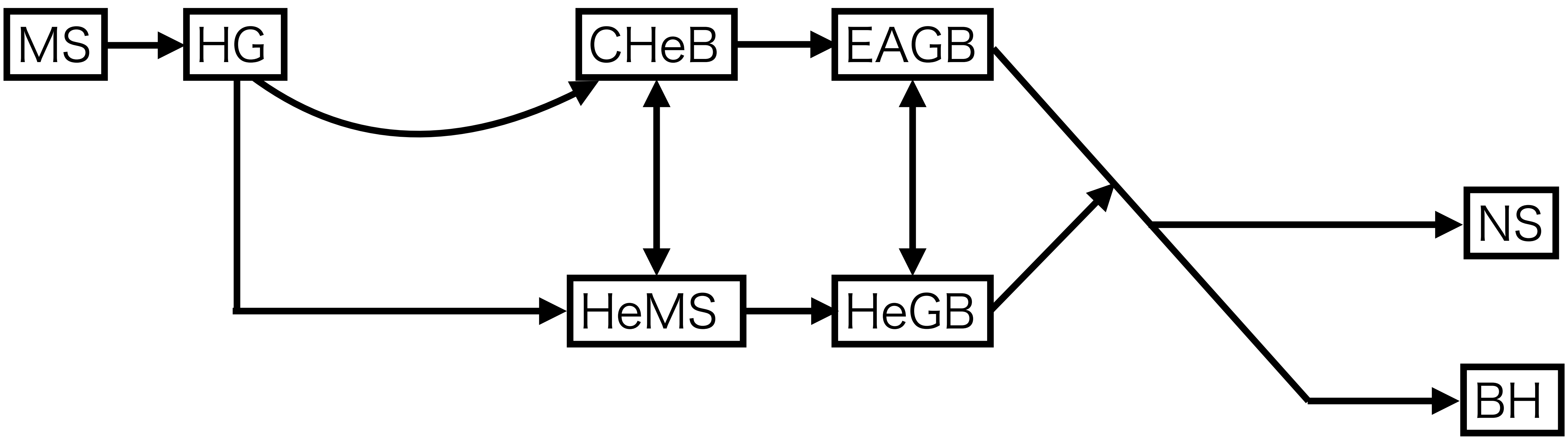
FF for each stellar type

- Main Sequence (MS)
- Hetzsprung Gap (HG)
- Giant Branch (GB)
- Core Helium Burning (CHeB)
- Early Asymptotic Giant Branch (EAGB)
- Thermally Pulsating AGB (TPAGB)
- (Supernova)
- (Remnant, such as white dwarf, neutron star, and black hole)

Evolution of stellar type



Evolution of stellar type



MSフィッティング公式(1)

On the MS we define a fractional time-scale

$$\tau = \frac{t}{t_{\rm MS}}. \quad (11)$$

As a star evolves across the MS, its evolution accelerates so that it is possible to model the time dependence of the logarithms of the luminosity and radius by polynomials in τ. Luminosity is given by

$$\log \frac{L_{\rm MS}(t)}{L_{\rm ZAMS}} = \alpha_L \tau + \beta_L \tau^{\eta} + \left(\log \frac{L_{\rm TMS}}{L_{\rm ZAMS}} - \alpha_L - \beta_L \right) \tau^2 - \Delta L (\tau_1^2 - \tau_2^2) \quad (12)$$

and radius by

$$\log \frac{R_{\rm MS}(t)}{R_{\rm ZAMS}} = \alpha_R \tau + \beta_R \tau^{10} + \gamma \tau^{40} + \left(\log \frac{R_{\rm TMS}}{R_{\rm ZAMS}} - \alpha_R - \beta_R - \gamma \right) \tau^3 - \Delta R (\tau_1^3 - \tau_2^3), \quad (13)$$

where

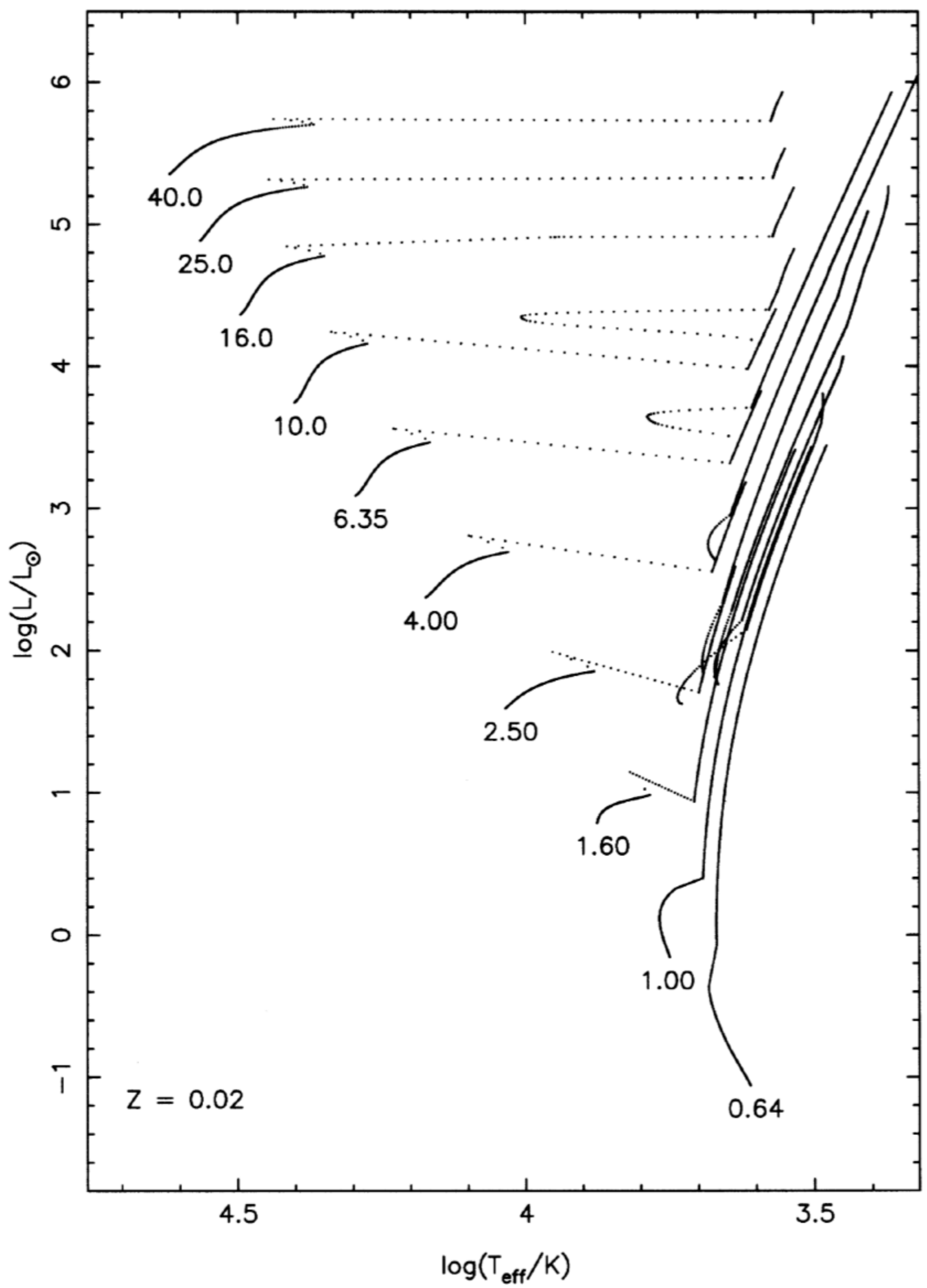
$$\tau_1 = \min(1.0, t/t_{\rm hook}) \quad (14)$$

$$\tau_2 = \max\left\{0.0, \min\left[1.0, \frac{t - (1.0 - \epsilon)t_{\rm hook}}{\epsilon t_{\rm hook}}\right]\right\} \quad (15)$$

for $\epsilon = 0.01$.

τ =0でZAMS,
τ =1でTMSに一致

ΔL, ΔRはhook starに対応



MSフィッティング公式(2)

On the MS we define a fractional time-scale

$$\tau = \frac{t}{t_{\rm MS}}. \tag{11}$$

As a star evolves across the MS, its evolution accelerates so that it is possible to model the time dependence of the logarithms of the luminosity and radius by polynomials in τ. Luminosity is given by

$$\log \frac{L_{\rm MS}(t)}{L_{\rm ZAMS}} = \alpha_L \tau + \beta_L \tau^{\eta} + \left(\log \frac{L_{\rm TMS}}{L_{\rm ZAMS}} - \alpha_L - \beta_L \right) \tau^2 - \Delta L (\tau_1^2 - \tau_2^2) \tag{12}$$

and radius by

$$\log \frac{R_{\rm MS}(t)}{R_{\rm ZAMS}} = \alpha_R \tau + \beta_R \tau^{10} + \gamma \tau^{40} + \left(\log \frac{R_{\rm TMS}}{R_{\rm ZAMS}} - \alpha_R - \beta_R - \gamma \right) \tau^3 - \Delta R (\tau_1^3 - \tau_2^3), \tag{13}$$

where

$$\tau_1 = \min(1.0, t/t_{\rm hook}) \tag{14}$$

$$\tau_2 = \max\left\{0.0, \min\left[1.0, \frac{t - (1.0 - \epsilon) t_{\rm hook}}{\epsilon t_{\rm hook}}\right]\right\} \tag{15}$$

for $\epsilon = 0.01$.

τ =0でZAMS,
τ =1でTMSに一致

ΔL, ΔRはhook
starに対応

The exponent $\eta = 10$ in equation (12), unless $Z \le 0.0009$ when it is given by

$$\eta = \begin{cases} 10 & M \le 1.0 \\ 20 & M \ge 1.1, \end{cases} \tag{18}$$

with linear interpolation between the mass limits.

detailed model is illustrated by Fig. 6. The luminosity pertubation is approximated by

$$\Delta L = \begin{cases} 0.0 & M \le M_{\rm hook} \\ B \left(\frac{M - M_{\rm hook}}{a_{33} - M_{\rm hook}} \right)^{0.4} & M_{\rm hook} < M < a_{33} \\ \min\left(\frac{a_{34}}{M^{a_{35}}}, \frac{a_{36}}{M^{a_{37}}} \right) & M \ge a_{33} \end{cases} \tag{16}$$

where $B = \Delta L(a_{33})$, $1.25 \le a_{33} \le 1.6$, $a_{35} \approx 0.4$ and $a_{37} \approx 0.6$.

The radius pertubation is approximated by

$$\Delta R = \begin{cases} 0.0 & M \le M_{\rm hook} \\ a_{43} \left(\frac{M - M_{\rm hook}}{a_{42} - M_{\rm hook}} \right)^{0.5} & M_{\rm hook} < M \le a_{42} \\ a_{43} + (B - a_{43}) \left(\frac{M - a_{42}}{2.0 - a_{42}} \right)^{a_{44}} & a_{42} < M < 2.0 \\ \frac{a_{38} + a_{39} M^{3.5}}{a_{40} M^3 + M^{a_{41}}} - 1.0 & M \ge 2.0, \end{cases} \tag{17}$$

MSフィッティング公式(3)

On the MS we define a fractional time-scale

$$\tau = \frac{t}{t_{\rm MS}}. \tag{11}$$

As a star evolves across the MS, its evolution accelerates so that it is possible to model the time dependence of the logarithms of the luminosity and radius by polynomials in τ. Luminosity is given by

$$\log \frac{L_{\rm MS}(t)}{L_{\rm ZAMS}} = \alpha_L \tau + \beta_L \tau^{\eta} + \left(\log \frac{L_{\rm TMS}}{L_{\rm ZAMS}} - \alpha_L - \beta_L \right) \tau^2 - \Delta L(\tau_1^2 - \tau_2^2) \tag{12}$$

and radius by

$$\log \frac{R_{\rm MS}(t)}{R_{\rm ZAMS}} = \alpha_R \tau + \beta_R \tau^{10} + \gamma \tau^{40} + \left(\log \frac{R_{\rm TMS}}{R_{\rm ZAMS}} - \alpha_R - \beta_R - \gamma \right) \tau^3 - \Delta R(\tau_1^3 - \tau_2^3), \tag{13}$$

τ =0でZAMS,
τ =1でTMSに一致

ΔL, ΔRはhook starに対応

where

$$\tau_1 = \min(1.0, t/t_{\rm hook}) \tag{14}$$

$$\tau_2 = \max\left\{0.0, \min\left[1.0, \frac{t - (1.0 - \epsilon)t_{\rm hook}}{\epsilon t_{\rm hook}}\right]\right\} \tag{15}$$

for $\epsilon = 0.01$.

$$\alpha_{\rm L} = \frac{a_{45} + a_{46}M^{a_{48}}}{M^{0.4} + a_{47}M^{1.9}} \qquad M \geq 2.0, \tag{19a}$$

The luminosity β coefficient is approximated by

$$\beta_L = \max(0.0, \ \ a_{54} - a_{55}M^{a_{56}}), \tag{20}$$

where $a_{56} \approx 0.96$. Then, if $M > a_{57}$ and $\beta_L > 0.0$,

$$\beta_L = \max(0.0, B - 10.0(M - a_{57})B),$$

where $B = \beta_L(M = a_{57})$ and $1.25 \leq a_{57} \leq 1.6$.

MSフィッティング公式(4)

On the MS we define a fractional time-scale

$$\tau = \frac{t}{t_{\rm MS}}. \qquad (11)$$

As a star evolves across the MS, its evolution accelerates so that it is possible to model the time dependence of the logarithms of the luminosity and radius by polynomials in τ. Luminosity is given by

$$\log \frac{L_{\rm MS}(t)}{L_{\rm ZAMS}} = \alpha_L \tau + \beta_L \tau^{\eta} + \left(\log \frac{L_{\rm TMS}}{L_{\rm ZAMS}} - \alpha_L - \beta_L\right)\tau^2 - \Delta L(\tau_1^2 - \tau_2^2) \qquad (12)$$

and radius by

$$\log \frac{R_{\rm MS}(t)}{R_{\rm ZAMS}} = \alpha_R \tau + \beta_R \tau^{10} + \gamma \tau^{40} + \left(\log \frac{R_{\rm TMS}}{R_{\rm ZAMS}} - \alpha_R - \beta_R - \gamma\right)\tau^3 - \Delta R(\tau_1^3 - \tau_2^3), \qquad (13)$$

where

$$\tau_1 = \min(1.0, t/t_{\rm hook}) \qquad (14)$$

$$\tau_2 = \max\left\{0.0, \min\left[1.0, \frac{t - (1.0-\epsilon)t_{\rm hook}}{\epsilon t_{\rm hook}}\right]\right\} \qquad (15)$$

for $\epsilon = 0.01$.

$τ$=0でZAMS,
$τ$=1でTMSに一致

$Δ$L, $Δ$Rはhook starに対応

$$\alpha_R = \begin{cases} a_{62} & M < 0.5 \\ a_{62} + (a_{63} - a_{62})(M - 0.5)/0.15 & 0.5 \le M < 0.65 \\ a_{63} + (a_{64} - a_{63})(M - 0.65)/(a_{68} - 0.65) & 0.65 \le M < a_{68} \\ a_{64} + (B - a_{64})(M - a_{68})/(a_{66} - a_{68}) & a_{68} \le M < a_{66} \\ C + a_{65}(M - a_{67}) & a_{67} < M, \end{cases} \qquad (21b)$$

where $B = \alpha_R(M = a_{66})$, $C = \alpha_R(M = a_{67})$, $0.8 \le a_{66} \le 1.6$, $3.5 \le a_{67} \le 7.2$ and $0.8 \le a_{66} \le 1.0$.

The radius β coefficient is approximated by $\beta_R = \beta'_R - 1$, where

$$\beta'_R = \frac{a_{69} M^{3.5}}{a_{70} + M^{a_{71}}} \qquad 2.0 \le M \le 16.0, \qquad (22a)$$

with $a_{71} \approx 3.5$ and $1.4 \le a_{74} \le 1.6$, and then

$$\beta'_R = \begin{cases} 1.06 & M \le 1.0 \\ 1.06 + (a_{72} - 1.06)(M - 1.0)/(a_{74} - 1.06) & 1.0 < M < a_{74} \\ a_{72} + (B - a_{72})(M - a_{74})/(2.0 - a_{74}) & a_{74} \le M < 2.0 \\ C + a_{73}(M - 16.0) & 16.0 < M \end{cases} \qquad (22b)$$

where $B = \beta'_R(M = 2.0)$, $C = \beta'_R(M = 16.0)$.

$$\gamma = \begin{cases} a_{76} + a_{77}(M - a_{78})^{a_{79}} & M \le 1.0 \\ B + (a_{80} - B)\left(\frac{M - 1.0}{a_{75} - 1.0}\right)^{a_{81}} & 1.0 < M \le a_{75} \\ C - 10.0(M - a_{75})C & a_{75} < M < a_{75} + 0.1, \end{cases} \qquad (23)$$

where $a_{79} \approx 9.4$, $a_{81} \approx 2.5$, $B = \gamma(M = 1.0)$ and $C = a_{80}$, unless $a_{75} = 1.0$ when $C = B$.

HGフィッティング公式

- HGのタイムスケールで規格化した時刻τの簡単な関数
- L_{TMS}とL_{EHG}, R_{TMS}とR_{EHG}, $M_{c,TMS}$と$M_{c,EHG}$をτの関数として繋ぐ
- $M_{c,TMS}$はρと$M_{c,EHG}$から得られる
- L_{TMS}, L_{EHG}, R_{TMS}, R_{EHG}, $M_{c,EHG}$は恒星進化コードから求める

5.1.2 Hertzsprung-gap evolution

During the HG we define

$$\tau = \frac{t - t_{MS}}{t_{BGB} - t_{MS}}. \tag{25}$$

Then for the luminosity and radius we simply take

$$L_{HG} = L_{TMS} \left(\frac{L_{EHG}}{L_{TMS}} \right)^{\tau} \tag{26}$$

$$R_{HG} = R_{TMS} \left(\frac{R_{EHG}}{R_{TMS}} \right)^{\tau}. \tag{27}$$

On the MS we do not consider the core to be dense enough with respect to the envelope to actually define a core mass, i.e., $M_{c,MS} = 0.0$. The core mass at the end of the HG is

$$M_{c,EHG} = \begin{cases} M_{c,GB}(L = L_{BGB}) & M < M_{HeF} \\ M_{c,BGB} & M_{HeF} \leq M < M_{FGB} \\ M_{c,HeI} & M \geq M_{FGB}, \end{cases} \tag{28}$$

where $M_{c,GB}$, $M_{c,BGB}$ and $M_{c,HeI}$ will be defined in Sections 5.2 and 5.3. At the beginning of the HG we set $M_{c,TMS} = \rho M_{c,EHG}$, where

$$\rho = \frac{1.586 + M^{5.25}}{2.434 + 1.02 M^{5.25}}, \tag{29}$$

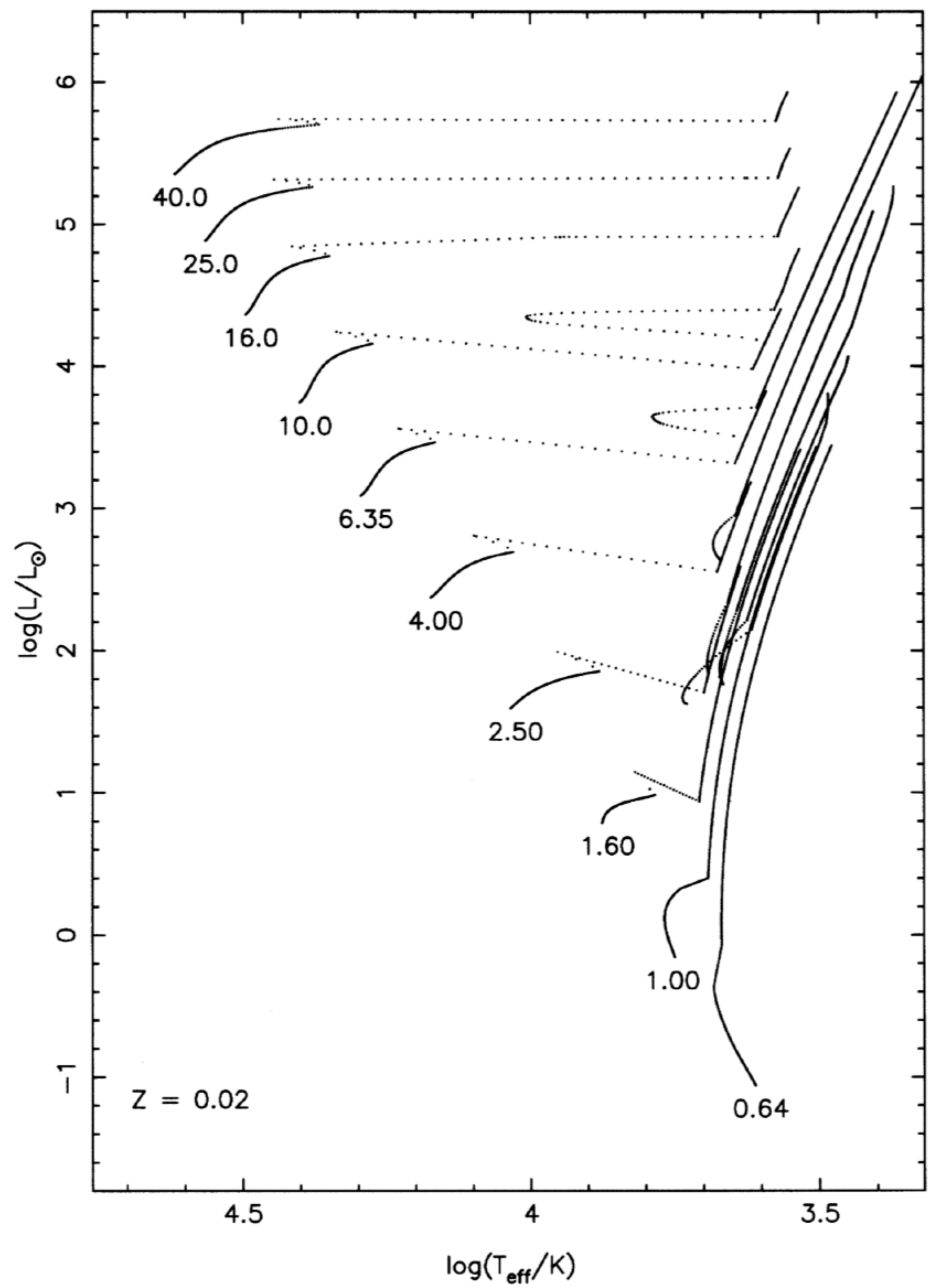
and simply allow the core mass to grow linearly with time so that

$$M_{c,HG} = [(1 - \tau)\rho + \tau] M_{c,EHG}. \tag{30}$$

If the HG star is losing mass (as described in Section 7.1), it is necessary to take $M_{c,HG}$ as the maximum of the core mass at the previous time-step and the value given by equation (30).

CHeB

- HM starの場合
- 始めはblueで，途中でredになる
  - redとはGBに乗ること
  - SSEでは必ず途中でredになる
- blueとredの場合のフィッティング公式



CHeBフィッティング公式(1)

Lについて

τはtHeで規格化した時刻

Then the luminosity during CHeB is modelled as

$$L = \begin{cases} L_x \left(\dfrac{L_{\rm BAGB}}{L_x} \right)^{\lambda} & \tau_x \le \tau \le 1 \\ L_x \left(\dfrac{L_{\rm HeI}}{L_x} \right)^{\lambda'} & 0 \le \tau < \tau_x \end{cases} \quad (61)$$

τxはblueが始まる時刻，すなわちτx=0

where

$$\lambda = \left(\frac{\tau - \tau_x}{1 - \tau_x} \right)^{\xi}; \quad \xi = \min[2.5, \max(0.4, R_{\rm mHe}/R_x)], \quad (62)$$

$$\lambda' = \left(\frac{\tau_x - \tau}{\tau_x} \right)^3. \quad (63)$$

Lxはblueが始まる時のL

$$L_x = \begin{cases} L_{\rm ZAHB} & M < M_{\rm HeF} \\ L_{\rm min,He} & M_{\rm HeF} \le M < M_{\rm FGB} \\ L_{\rm HeI} & M \ge M_{\rm FGB} \end{cases} \quad (59)$$

The lifetime of CHeB is given by

$$t_{\rm He} = \begin{cases} \{b_{39} + [t_{\rm HeMS}(M_c) - b_{39}](1-\mu)^{b_{40}}\} \\ \quad \times [1 + \alpha_4 \exp 15(M - M_{\rm HeF})] & M < M_{\rm HeF} \\ t_{\rm BGB} \dfrac{b_{41} M^{b_{42}} + b_{43} M^5}{b_{44} + M^5} & M \ge M_{\rm HeF} \end{cases} \quad (57)$$

$$R_{\rm mHe} = \frac{b_{24} M + (b_{25} M)^{b_{26}} M^{b_{28}}}{b_{27} + M^{b_{28}}} \qquad M \ge M_{\rm HeF}. \quad (55)$$

$$R_x = \begin{cases} R_{\rm ZAHB} & M < M_{\rm HeF} \\ R_{\rm GB}(L_{\rm min,He}) & M_{\rm HeF} \le M < M_{\rm FGB} \\ R_{\rm HeI} & M \ge M_{\rm FGB} \end{cases}. \quad (60)$$

CHeBフィッティング公式(2)

Rについて

The *actual* minimum radius during CHeB is $R_{\rm min} = \min(R_{\rm mHe}, R_x)$, because equation (55) for $R_{\rm mHe}$ can give a value that is greater than R_x (this property is used, however, to compute ξ above). Furthermore, we define τ_y as the relative age at the end of the blue phase of CHeB, and L_y and R_y as the luminosity and radius at $\tau = \tau_y$. Hence $\tau_y = 1$ for LM and IM stars, and $\tau_y = \tau_{\rm bl}$ for HM stars. L_y is given by equation (61) ($L_y = L_{\rm BAGB}$ for $M \leq M_{\rm FGB}$), and $R_y = R_{\rm AGB}(L_y)$. The radius during CHeB is modelled as

$$R = \begin{cases} R_{\rm GB}(M, L) & 0 \leq \tau < \tau_x \\ R_{\rm AGB}(M, L) & \tau_y < \tau \leq 1 \\ R_{\rm min} \exp(|\rho|^3) & \tau_x \leq \tau \leq \tau_y \end{cases} \quad (64)$$

where

$$\rho = \left(\ln \frac{R_y}{R_{\rm min}}\right)^{\frac{1}{3}} \left(\frac{\tau - \tau_x}{\tau_y - \tau_x}\right) - \left(\ln \frac{R_x}{R_{\rm min}}\right)^{\frac{1}{3}} \left(\frac{\tau_y - \tau}{\tau_y - \tau_x}\right). \quad (65)$$

τ yはblueが終わる時刻

RyはAGBでの半径の計算から得られる値

The lifetime of CHeB is given by

$$t_{\rm He} = \begin{cases} \{b_{39} + [t_{\rm HeMS}(M_{\rm c}) - b_{39}](1-\mu)^{b_{40}}\} \\ \quad \times [1 + \alpha_4 \exp 15(M - M_{\rm HeF})] & M < M_{\rm HeF} \\ t_{\rm BGB} \dfrac{b_{41} M^{b_{42}} + b_{43} M^5}{b_{44} + M^5} & M \geq M_{\rm HeF} \end{cases} \quad (57)$$

with $\alpha_4 = [t_{\rm He}(M_{\rm HeF}) - b_{39}]/b_{39}$. The term involving $t_{\rm HeMS}(M_{\rm c})$ ensures continuity with the lifetime of a naked helium star with $M = M_{\rm c}$ as the envelope mass vanishes. The lifetime of the blue phase of CHeB relative to $t_{\rm He}$ depends in a complicated way on M and Z; it is roughly approximated by

$$\tau_{\rm bl} = \begin{cases} 1 & M < M_{\rm HeF} \\ b_{45} \left(\dfrac{M}{M_{\rm FGB}}\right)^{0.414} + \alpha_{\rm bl} \left(\log \dfrac{M}{M_{\rm FGB}}\right)^{b_{46}} & M_{\rm HeF} \leq M \leq M_{\rm FGB} \\ (1 - b_{47}) \dfrac{f_{\rm bl}(M)}{f_{\rm bl}(M_{\rm FGB})} & M > M_{\rm FGB}, \end{cases} \quad (58)$$

truncated if necessary to give $0 \leq \tau_{\rm bl} \leq 1$, where

$$\alpha_{\rm bl} = \left[1 - b_{45} \left(\frac{M_{\rm HeF}}{M_{\rm FGB}}\right)^{0.414}\right] \left(\log \frac{M_{\rm HeF}}{M_{\rm FGB}}\right)^{-b_{46}}$$

and

$$f_{\rm bl}(M) = M^{b_{48}} \left\{1 - \frac{R_{\rm mHe}(M)}{R_{\rm AGB}[L_{\rm HeI}(M)]}\right\}^{b_{49}}.$$

MFGBでのRmHeやLHeIを知る必要がある気が，，，

CHeBフィッティング公式(3)

Mcについて

The core mass $M_{\mathrm{c,HeI}}$ at helium ignition is given by the M_{c}–L relation for LM stars, while for $M \geq M_{\mathrm{HeF}}$ the same formula can be used as for the BGB core mass (equation 44), replacing $M_{\mathrm{c}}[L_{\mathrm{BGB}}(M_{\mathrm{HeF}})]$ with $M_{\mathrm{c}}[L_{\mathrm{HeI}}(M_{\mathrm{HeF}})]$ to ensure continuous transition at $M = M_{\mathrm{HeF}}$. For $M > 3\,\mathrm{M}_{\odot}$, $M_{\mathrm{c,HeI}}$ is nearly equal to $M_{\mathrm{c,BGB}}$. The core mass at the BAGB point is approximated by

$$M_{\mathrm{c,BAGB}} = (b_{36} M^{b_{37}} + b_{38})^{\frac{1}{4}}, \tag{66}$$

where $b_{36} \approx 4.36 \times 10^{-4}$, $b_{37} \approx 5.22$ and $b_{38} \approx 6.84 \times 10^{-2}$. In between, the core mass is taken to simply increase linearly with time:

$$M_{\mathrm{c}} = (1 - \tau) M_{\mathrm{c,HeI}} + \tau M_{\mathrm{c,BAGB}}. \tag{67}$$

EAGBとTPAGB

- EAGB … HeコアがCOコアに変換されるフェイズ
- TPAGB … HeコアがCOコアに完全に変換されたあとのフェイズ
- HM star ($M>M_{FAGB}$)ではEAGB段階でC ignitionが起こるのでTPAGBは考えなくて良い
- フィッティングの仕方はGBと同じ

EAGBフィッティング公式(1)

EggletonのMc-L関係

$$L = DM_c^p. \qquad (31)$$

McはCOコアの質量

The evolution is then determined by the growth of the core mass as a result of hydrogen burning which, in a state of thermal equilibrium, is given by

$$L = EX_e\dot{M}_c \Rightarrow \dot{M}_c = A_H L, \qquad (32)$$

where

X_e = envelope mass fraction of hydrogen,

E = the specific energy release and

A_H = hydrogen rate constant.

Thus

$$\frac{dM_c}{dt} = A_H DM_c^p, \qquad (33)$$

which upon integration gives

$$M_c = [(p-1)A_H D(t_{inf} - t)]^{\frac{1}{1-p}} \qquad (34)$$

or

$$L = D[(p-1)A_H D(t_{inf} - t)]^{\frac{p}{1-p}}, \qquad (35)$$

so that the time evolution of either M_c or L is given and we can then simply find the other from the M_c–L relation. Also, when $L = L_{BGB}$ we have $t = t_{BGB}$, which defines the integration constant

$$t_{inf} = t_{BGB} + \frac{1}{A_H D(p-1)}\left(\frac{D}{L_{BGB}}\right)^{\frac{p-1}{p}}. \qquad (36)$$

実際にはdouble power law

$$L = \min(BM_c^q, DM_c^p) \qquad (q < p), \qquad (37)$$

$$M_x = \left(\frac{B}{D}\right)^{\frac{1}{p-q}}. \qquad (38)$$

$$M_{c,GB} = \begin{cases} [(p-1)A_H D(t_{inf,1} - t)]^{\frac{1}{1-p}} & t \le t_x \\ [(q-1)A_H B(t_{inf,2} - t)]^{\frac{1}{1-q}} & t > t_x \end{cases} \qquad (39)$$

for $t_{BGB} \le t \le t_{HeI}$, where

$$t_{inf,1} = t_{BGB} + \frac{1}{(p-1)A_H D}\left(\frac{D}{L_{BGB}}\right)^{\frac{p-1}{p}} \qquad (40)$$

$$t_x = t_{inf,1} - (t_{inf,1} - t_{BGB})\left(\frac{L_{BGB}}{L_x}\right)^{\frac{p-1}{p}} \qquad (41)$$

$$t_{inf,2} = t_x + \frac{1}{(q-1)A_H B}\left(\frac{B}{L_x}\right)^{\frac{q-1}{q}}. \qquad (42)$$

The GB ends at $t = t_{HeI}$, corresponding to $L = L_{HeI}$ (see Section 5.3), given by

$$t_{HeI} = \begin{cases} t_{inf,1} - \frac{1}{(p-1)A_H D}\left(\frac{D}{L_{HeI}}\right)^{\frac{p-1}{p}} & L_{HeI} \le L_x \\ t_{inf,2} - \frac{1}{(q-1)A_H B}\left(\frac{B}{L_{HeI}}\right)^{\frac{q-1}{q}} & L_{HeI} > L_x \end{cases}. \qquad (43)$$

EAGBフィッティング公式(2)

M, Z依存性

$$p = \begin{cases} 6 & M \le M_{\rm HeF} \\ 5 & M \ge 2.5 \end{cases}$$

$$q = \begin{cases} 3 & M \le M_{\rm HeF} \\ 2 & M \ge 2.5 \end{cases}$$

$$B = \max(3\times10^4, 500 + 1.75\times10^4 M^{0.6})$$

$$\log D = \begin{cases} 5.37 + 0.135\zeta \quad [= D_0] & M \le M_{\rm HeF} \\ \max(-1.0, 0.975D_0 - 0.18M, 0.5D_0 - 0.06M) & \\ & M \ge 2.5 \end{cases}$$

AH->AHe

and be converted to oxygen. Thus

$$E = \frac{\epsilon_{3\alpha} + 0.75\epsilon_{C\alpha}}{15m(H)} \approx 8.09\times10^{17}\,{\rm erg\,g^{-1}},$$

so that

$$A_{\rm He} = (EX_{\rm He})^{-1} = 7.66\times10^{-5}\,{\rm M_\odot\,L_\odot^{-1}\,Myr^{-1}} \quad (68)$$

using $X_{\rm He} \approx 0.98$. Although massive stars ($M \gtrsim 8$) do not

実際にはdouble power law

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$$M_x = \left(\frac{B}{D}\right)^{\frac{1}{p-q}}. \quad (38)$$

$$M_{\rm c,GB} = \begin{cases} [(p-1)A_{\rm H}D(t_{\rm inf,1} - t)]^{\frac{1}{1-p}} & t \le t_x \\ [(q-1)A_{\rm H}B(t_{\rm inf,2} - t)]^{\frac{1}{1-q}} & t > t_x \end{cases} \quad (39)$$

for $t_{\rm BGB} \le t \le t_{\rm HeI}$, where

$$t_{\rm inf,1} = t_{\rm BGB} + \frac{1}{(p-1)A_{\rm H}D}\left(\frac{D}{L_{\rm BGB}}\right)^{\frac{p-1}{p}} \quad (40)$$

$$t_x = t_{\rm inf,1} - (t_{\rm inf,1} - t_{\rm BGB})\left(\frac{L_{\rm BGB}}{L_x}\right)^{\frac{p-1}{p}} \quad (41)$$

$$t_{\rm inf,2} = t_x + \frac{1}{(q-1)A_{\rm H}B}\left(\frac{B}{L_x}\right)^{\frac{q-1}{q}}. \quad (42)$$

The GB ends at $t = t_{\rm HeI}$, corresponding to $L = L_{\rm HeI}$ (see Section 5.3), given by

$$t_{\rm HeI} = \begin{cases} t_{\rm inf,1} - \frac{1}{(p-1)A_{\rm H}D}\left(\frac{D}{L_{\rm HeI}}\right)^{\frac{p-1}{p}} & L_{\rm HeI} \le L_x \\ t_{\rm inf,2} - \frac{1}{(q-1)A_{\rm H}B}\left(\frac{B}{L_{\rm HeI}}\right)^{\frac{q-1}{q}} & L_{\rm HeI} > L_x \end{cases}. \quad (43)$$

EAGBフィッティング公式(3)

Rの決め方

The radius evolution is very similar to that of the GB, as the stars still have a deep convective envelope, but with some slight modifications. The basic formula is the same,

$$R_{\rm AGB} = A(L^{b_1} + b_2 L^{b_{50}}). \tag{74}$$

where indeed b_1 and b_2 are exactly the same as for $R_{\rm GB}$, and $b_{50} = b_{55} b_3$ for $M \geq M_{\rm HeF}$. Also, for $M \geq M_{\rm HeF}$,

$$A = \min(b_{51} M^{-b_{52}}, b_{53} M^{-b_{54}}),$$

which gives

$$R_{\rm AGB} = 1.125 M^{-0.33}(L^{0.4} + 0.383 L^{0.76}),$$

as an example, for $Z = 0.02$. For $M < M_{\rm HeF}$ the behaviour is slightly altered, so we take

$$b_{50} = b_3$$

$$A = b_{56} + b_{57} M$$

for $M \leq M_{\rm HeF} - 0.2$ and linear interpolation between the bounding values for $M_{\rm HeF} - 0.2 < M < M_{\rm HeF}$, which means that for $M = 1.0$ and $Z = 0.02$ the relation gives

$$R_{\rm AGB} \approx 0.95(L^{0.4} + 0.383 L^{0.74}).$$

実際にはdouble power law

$$L = \min(B M_{\rm c}^q, D M_{\rm c}^p) \qquad (q < p), \tag{37}$$

$$M_x = \left(\frac{B}{D}\right)^{\frac{1}{p-q}}. \tag{38}$$

$$M_{\rm c,GB} = \begin{cases} [(p-1) A_{\rm H} D (t_{\rm inf,1} - t)]^{\frac{1}{1-p}} & t \leq t_x \\ [(q-1) A_{\rm H} B (t_{\rm inf,2} - t)]^{\frac{1}{1-q}} & t > t_x \end{cases} \tag{39}$$

for $t_{\rm BGB} \leq t \leq t_{\rm HeI}$, where

$$t_{\rm inf,1} = t_{\rm BGB} + \frac{1}{(p-1) A_{\rm H} D}\left(\frac{D}{L_{\rm BGB}}\right)^{\frac{p-1}{p}} \tag{40}$$

$$t_x = t_{\rm inf,1} - (t_{\rm inf,1} - t_{\rm BGB})\left(\frac{L_{\rm BGB}}{L_x}\right)^{\frac{p-1}{p}} \tag{41}$$

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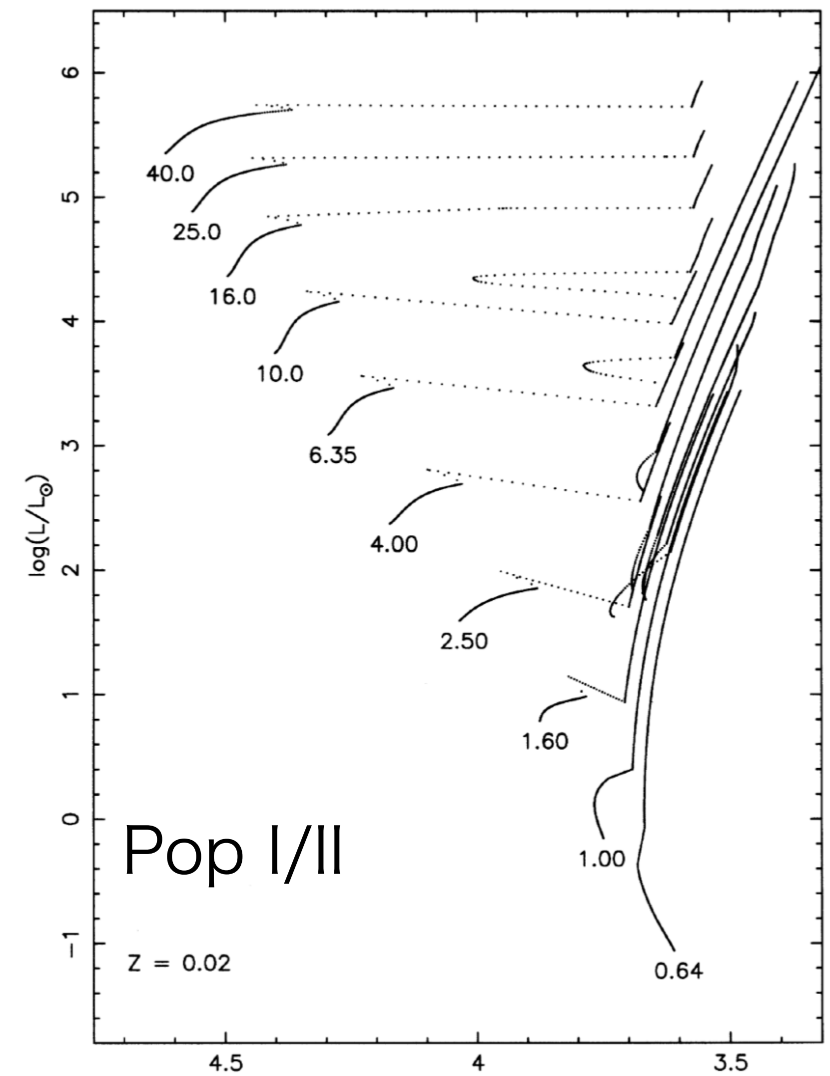
The GB ends at $t = t_{\rm HeI}$, corresponding to $L = L_{\rm HeI}$ (see Section 5.3), given by

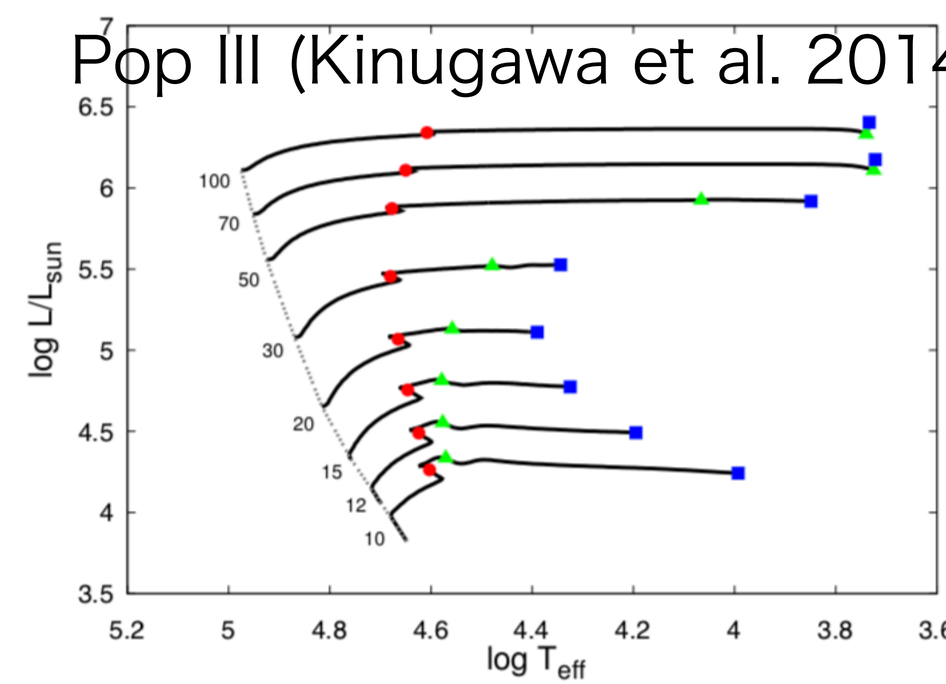
$$t_{\rm HeI} = \begin{cases} t_{\rm inf,1} - \frac{1}{(p-1) A_{\rm H} D}\left(\frac{D}{L_{\rm HeI}}\right)^{\frac{p-1}{p}} & L_{\rm HeI} \leq L_x \\ t_{\rm inf,2} - \frac{1}{(q-1) A_{\rm H} B}\left(\frac{B}{L_{\rm HeI}}\right)^{\frac{q-1}{q}} & L_{\rm HeI} > L_x \end{cases}. \tag{43}$$

Problem of Hurley's FF

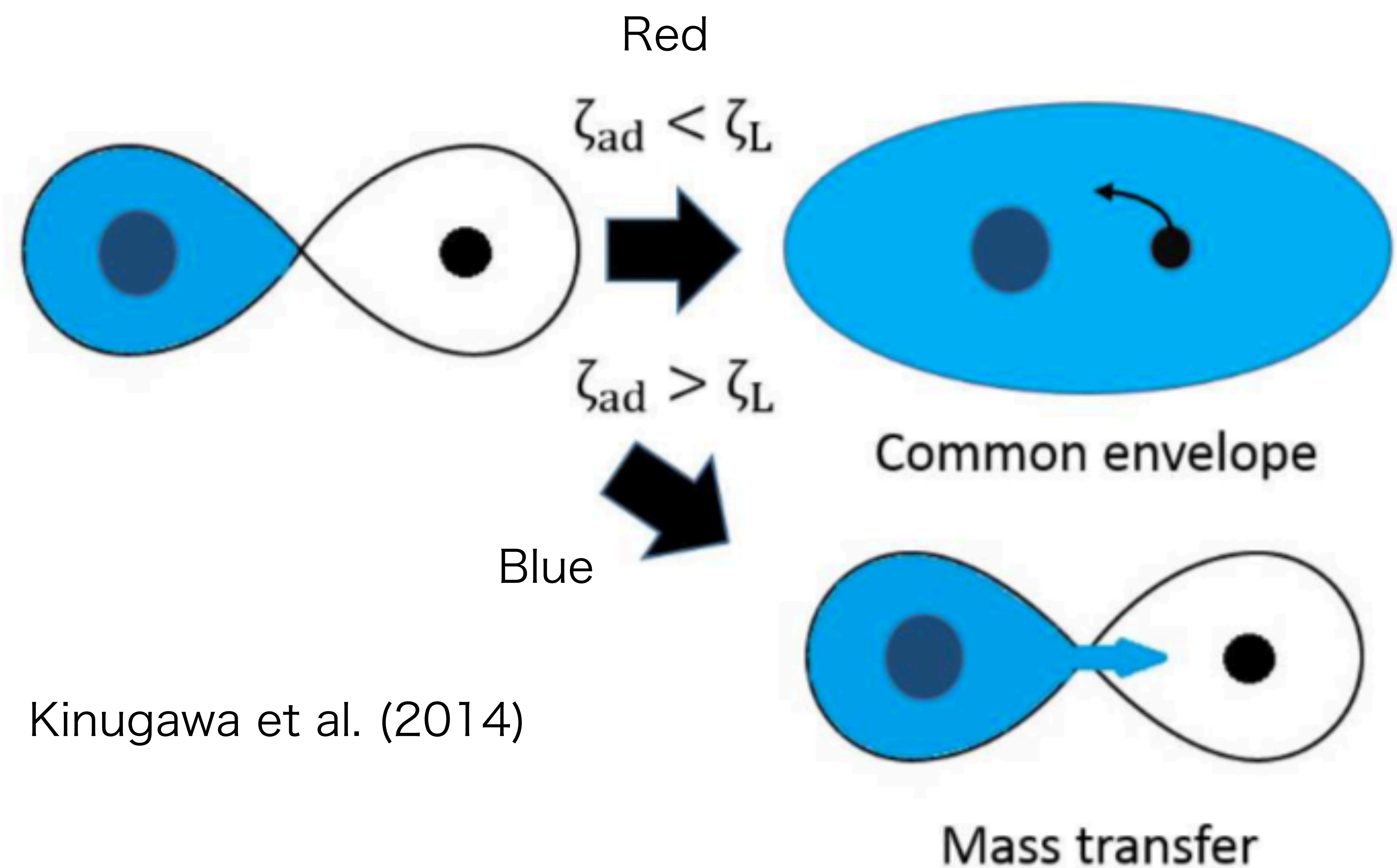
- CHeB stars enter into GB necessarily.
- But, EMP stars do not necessarily enter into GB until their evolution finish.
- Since Hurley's code is intensively tuned, it is difficult to adapt to EMP stars.
- FFs like Kinugawa's FF will be fit to Kinugawa's code easily.



Pop III (Kinugawa et al. 2014)



Problem of Hurley's FF



Kinugawa et al. (2014)

Summary

- We will make FFs similarly to Kinugawa's FF.
- We have got stellar evolution data of $Z/Z_\odot=10^{-8}$ made by Takashi Yoshida.
- First, we make FFs for $Z/Z_\odot=10^{-8}$, and compare Kinugawa's FF.
- Next, we make FFs for other $Z/Z_\odot$.